PISCATAWAY TOWNSHIP SCHOOLS

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Essentials of Algebra 2

**Content Area:**	Mathematics
**Grade Span:**	11-12
**Revised by:**	Justin Cere, Mitchell Fuhr, Michelle Garlatti, Robin Styles
**Presented By**	Frank Wrublevski, Supervisor of Mathematics 7-12
**Approval Date:**	August 2023

COURSE OVERVIEW

Description
Essentials of Algebra 2 is a year-long course for upperclassmen, and the course is designed to develop a strong foundation in logical thinking and problem-solving that will enable students to make informed decisions regarding matters of money and finance in their daily lives. This course furthers the development of functions, which include linear, exponential, piece-wise, quadratics, and step functions. Other topics studied include measures of center and spread, graphical representations of data, principles of finance economics, amortization, revenue, and profit functions, loans, compound interest, continuous interest, car ownership, and budgets. The curriculum is a balance of traditional topics in Algebra 2 and thematic units to spark student interest and make math accessible and applicable to their life beyond graduation.

Goals
In addition to the content standards, skills, and concepts set forth, this course also seeks to meet the Standards for Mathematical Practice. These practices include generally applied best practices for learning mathematics, such as understanding the nature of proof and having a productive disposition towards the subject, and are not tied to a particular set of content. These skills are applicable beyond a student's study of mathematics. The eight Standards for Mathematical Practice are outlined below: 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Scope and Sequence

Unit	Topic	Length (90 Blocks)
1	Algebra Extension 1: Equations and Inequalities	8-10 Blocks
2	Taxes and Stock Market	8-9 Blocks
3	Algebra Extension 2: The Algebra of Geometry	12-14 Blocks
4	Banking & Credit	14-16 Blocks
5	Algebra Extension 3: Polynomials	12-14 Blocks
6	Automobile Unit	14-16 Blocks
7	Algebra Extension 4: Trigonometry	9-10 Blocks

Resources
Core Text: Gerver, Robert K., and Richard J. Sgroi. *Financial Algebra: Advanced Algebra with Financial Applications*. National Geographic Learning/Cengage, 2021. **Suggested Resources:** Desmos, Geogebra, Albert.io

UNIT 1: Algebra Extension 1: Equations and Inequalities

	Summary and Rationale

	Summary and Rationale	

<table>
<tr><td colspan="2" align="center">Summary and Rationale</td></tr>
<tr><td colspan="2">In this unit, students revisit strategies for solving one-variable equations, including making balanced moves and using inverse operations. Students represent situations using two variable equations and rewrite those equations to highlight a variable of interest. Students write inequalities in one and two variables to represent constraints. They also determine solutions to inequalities algebraically and graphically. Modeling and problem-solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.</td></tr>
<tr><td colspan="2" align="center">Recommended Pacing</td></tr>
<tr><td colspan="2" align="center">8-10 Blocks</td></tr>
<tr><td colspan="2" align="center">State Standards</td></tr>
<tr><td colspan="2">Standard A-CED Creating Equations</td></tr>
<tr><td>CPI #</td><td>Cumulative Progress Indicator (CPI)</td></tr>
<tr><td>1</td><td>Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions</td></tr>
<tr><td>2</td><td>Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</td></tr>
<tr><td>3</td><td>Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</td></tr>
<tr><td>4</td><td>Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.</td></tr>
<tr><td colspan="2">Standard A-REI Reasoning with Equations and Inequalities</td></tr>
<tr><td>CPI #</td><td>Cumulative Progress Indicator (CPI)</td></tr>
<tr><td>1</td><td>Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</td></tr>
<tr><td>3</td><td>Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</td></tr>
<tr><td>10</td><td>Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</td></tr>
<tr><td>11</td><td>Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.</td></tr>
<tr><td>12</td><td>Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</td></tr>
<tr><td colspan="2">Standard F-IF Interpreting Functions</td></tr>
<tr><td>CPI #</td><td>Cumulative Progress Indicator (CPI)</td></tr>
<tr><td>1</td><td>Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its</td></tr>
</table>

	domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*
5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F*or example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*.
9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Standard F-BF Building Functions

CPI #	Cumulative Progress Indicator (CPI)
1	Write a function that describes a relationship between two quantities.

Instructional Focus

Unit Enduring Understandings

- Equations and inequalities may be used as models to solve mathematical and real-world problems.
- Real-world problems may be represented by the formation and solution of linear equations.
- Variables represent one number and any given solution may be checked for precision.
- Real-world problems may be represented by the formation and solution of linear equations.
- An inequality is another way to describe a relationship between expressions; instead of showing that the values of two expressions are equal, inequalities indicate that the value of one expression is greater than (or greater than or equal to) the value of the other expression.
- In solving an inequality, multiplying or dividing both expressions by a negative number reverses the sign that indicates the relationships between the two expressions.

Unit Essential Questions

- How can we create an equation (or inequality) for a given situation?
- How does the way we represent a solution help us understand its meaning?
- How does the structure of a problem help us find a solution?
- What does it mean for a solution to be finite vs. infinite?
- How can we solve multi-step equations? How can we check that solution?

Objectives

Students will know:
- Linear equations can have one, none, or infinitely many solutions
- Multiple representations used to solve an equation
- Multi-variable equations can be solved for a given variable
- Solutions to inequality can be represented in multiple way

Vocabulary: Linear equations, slope, y-intercept, solution, inequality, intercepts, domain, range

Students will be able to:
- Describe and use balanced moves to solve an equation (Desmos 2.1-2.2 combined) (Block 1)
- Interpret and write one-variable inequalities that represent constraints. Desmos 2.10-2.11 combined) (Block 2)

- Solve one-variable inequalities by reasoning. (Desmos 2.10-2.11 combined) (Block 2)
- Graph solutions to a one-variable inequality on the number line (Desmos 2.10-2.11 combined) (Block 2)
- Write an equation of a line slope intercept, point slope, and standard form (Block 3)
- Graph equations in slope intercept, point slope, and standard form (Include horizontal and vertical lines) (Block 4)
- Identify parallel and perpendicular lines based on given equations (Block 4)
- Identify the domain and range of a given function using set builder and interval notation Desmos 4.8-4.9 combined) (Block 5)
- Identify the input and output of a function in proper function notation (Block 5)
- Classify a function given a picture of a graph (Block 6)
- Identify the shape of a graph given the equation of a function (Block 6)
- Solve and graph linear inequalities in the coordinate plane (Block7)
- Solve systems of linear equations with absolute value graphs and circles centered at the origin by graphing (Block 8)
- Solve systems of linear equations algebraically (Block 9)

Resources

Core Text: Gerver, Robert K., and Richard J. Sgroi. *Financial Algebra: Advanced Algebra with Financial Applications*. National Geographic Learning/Cengage, 2021.

Suggested Resources: Desmos, Geogebra, Albert.io

UNIT 2: Taxes and Stock Market

Summary and Rationale

The taxes and stock market unit will take a look at multiple mathematical concepts through the lens of tax forms and stock ticker information. Students will calculate owed income tax compared to previously paid tax, the total value of stock holdings, as well as net gains and losses from both individual stocks and total portfolios. Foundational skills for this unit will be inequalities, data displays (including piecewise graphs, bar charts, and candlestick charts), and arithmetic operations to complete spreadsheets. Students will build upon prior knowledge and skills in arithmetic and mathematics as they apply these concepts in real-world contexts. Students will have the opportunity to evaluate how the fields of psychology, government, sociology, business, and humanities impact real-world decision-making processes and how relevant quantitative and computational thinking is for long-term success in life.

Recommended Pacing

8-9 Blocks

State Standards

Standard N-Q Quantities

CPI #	Cumulative Progress Indicator (CPI)
1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2	Define appropriate quantities for the purpose of descriptive modeling.
3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Standard A-CED Creating Equations

CPI #	Cumulative Progress Indicator (CPI)
1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Standard F-IF

CPI #	Cumulative Progress Indicator (CPI)
8	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Standard A-REI

CPI #	Cumulative Progress Indicator (CPI)
12	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to

graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Standard S-ID

CPI #	Cumulative Progress Indicator (CPI)
1	Represent data with plots on the real number line (dot plots, histograms, and box plots).
5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related

Instructional Focus

Unit Enduring Understandings

- Income taxes are an aspect of society that require an understanding of practical math skills
- When filing taxes, there are a variety of strategies to support the process and make it as efficient and effective as possible.
- The stock market presents opportunities to make money; however, there are significant risks associated with managing money via stocks.

Unit Essential Questions

- Who pays taxes?
- How do you know how much to pay in taxes?
- How can you display tax owed in a graph?
- What does the information on a paystub, W-2, 1099 or 1040 mean? How do you use it?
- What is the difference between gross pay and net pay?
- How do you read stock data and stock candlestick charts to follow daily progress?
- What is a moving average and how is it different from a traditional average?
- How can you write a spreadsheet formula to calculate a moving average?
- How do you determine the total value invested into a stock?
- How can you compute your gains and losses in the stock market?

Objectives

Students will know:

- What a tax form looks like and how to fill it out.
- How to compute owed income taxes from a table.
- How to read a tax table and a tax graph.
- What the stock market is and the differences between NYSE, NASDAQ, DOW JONES, S&P500, etc.
- How to use spreadsheets to express data.
- The benefits of using a simple moving average.
- How to read stock ticker information and use it to calculate total value, gains, and losses.

Vocabulary: Property tax, sales tax, taxable income, income tax, Internal Revenue Service, single, head of household, constraint, inequality constraint, flat tax, proportional tax, progressive tax system, tax bracket, regressive tax schedule, gross pay, net pay, take-home pay, paycheck, Form W-4, withholding tax, Form W-2, Form 1099, tax-deferred contribution, flexible spending account, dependent, Form 1040, standard deduction, tax return, stock market, trades, NYSE, NASDAQ, net change, stock chart, stock bar chart, candlestick chart, simple moving average, total value of trade, net money flow, portfolio, gross capital gain, gross capital loss.

Students will be able to:

- Express tax schedules algebraically (6.1) (Block 1)
- Compute income taxes using a tax table(6.1) (Block 1)
- Construct income tax graphs using systems of equations/piecewise functions (6.2) (Block 2)
- Interpret and use the information on a paystub, W-2 and 1099 (6.3) (Block 3)
- Understand and fill out a 1040 to compute tax liability (6.4) (Block 4)
- Use stock data to follow the daily progress (8.2) (Block 5)
- Interpret a stock bar and stock candlestick charts (8.3) (Block 6)
- Write spreadsheet formulas (8.2) (Block 6)
- Calculate simple moving averages using arithmetic average formula/spreadsheet (8.4) (Block 7)
- Explain stock market ticker information and determine total value and volumes from information (8.5) (Block 7)
- Compute gains and losses (8.6) (Block 8)

Resources

Core Text: Gerver, Robert K., and Richard J. Sgroi. *Financial Algebra: Advanced Algebra with Financial Applications*. National Geographic Learning/Cengage, 2021.

Suggested Resources: Desmos, Geogebra, Albert.io

UNIT 3: Algebra Extension 2: The Algebra of Geometry

Summary and Rationale		
In this unit students revisit angles with parallel lines through transversals and solve for right triangle unknown sides and angles. The Unit continues by introducing the unit circle and its relation to trigonometry. This leads to exponential growth and decay to prepare the students for the banking and credit unit.		

Recommended Pacing		
12-14 blocks		

State Standards		

Standard G-CO Geometry

CPI #	Cumulative Progress Indicator (CPI)
11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals

Standard G-SRT

CPI #	Cumulative Progress Indicator (CPI)
1	Verify experimentally the properties of dilations given by a center and a scale factor:
1a	A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
1b	The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Standard G-GMD

CPI #	Cumulative Progress Indicator (CPI)
4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects

Standard A-CED

CPI #	Cumulative Progress Indicator (CPI)
1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

Standard G-GMD

CPI #	Cumulative Progress Indicator (CPI)
1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone
3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

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Standard F-IF Interpreting Functions	
CPI #	**Cumulative Progress Indicator (CPI)**
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. K
7c	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Instructional Focus

Unit Enduring Understandings

- The process of proving includes developing conjecture, considering a general case, exploring with examples, looking for structural similarities across cases, and searching for counterexamples.
- There are special relationships between the angles and sides of triangles, particularly right triangles. These relationships can be used to solve problems involving missing side lengths or angle measures.
- The value of a particular representation depends on its purpose.
- Characteristics depend on the situation.
- Perspective builds understanding.

Unit Essential Questions

- What is the relationship between the angles of a triangle and the side lengths of the triangle?
- How can trigonometry be used in indirect measurement or estimation?
- How is change related to behavior?
- Are characteristics unique?
- Can situations be represented graphically?

Objectives

Students will know:

- The relationship between the sides of special 30-60-90 and 45-45-90 right triangles
- There are six trigonometric ratio functions related to a right triangle
- Trig functions allow you to solve any right triangle
- Why surface area and volume are different and why they are important

  and their real-world applications

Vocabulary: parallel, perpendicular, transversal, complementary, supplementary, vertical, alternate exterior, alternate interior, corresponding angles, consecutive interior angles, scale factor, enlargement. reduction, dilation, hypotenuse, leg, 30-60-90 triangle, 45-45-90 triangle, opposite leg, adjacent leg, hypotenuse, sine, cosine, tangent, edges, vertices, faces, cross-section, pyramid, prism, cone, cylinder, sphere, surface area, net .lateral area, Euler's Formula volume, exponential function, initial value, growth rate, decay rate, increasing or decreasing function, positive or negative interval

Students will be able to:

- Identify pairs of angles formed by transversals and multiple intersecting lines (complementary, supplementary, vertical, alternate exterior, alternate interior, corresponding, (Block 1)
- Use properties of parallel lines to find angle measures. (Block 2)
- Informally prove theorems about parallel and perpendicular lines using algebra (Block 2)
- Informally prove theorems about identifying parallel lines.(Converse Theorems)(Algebraically) (Block 3)
- Identify scale factors and dilations in a coordinate plane. (Block 4)
- Use special right triangles to find missing lengths. (Block 5)

- Use the six trig functions to solve right triangles (Block 6)
- Use Inverse trig to solve for missing angles in right triangles (Block 6)
- Relationships between two and three-dimensional shapes (Block 7)
- Describe attributes of solids (Block 7)
- Describe and draw cross sections (Block 7)
- Solve a literal geometric equation (Block 8)
- Find surface areas and volumes of solids and composite solids (Prisms, cylinders, pyramids, cones, and spheres) (Block 9 &10)
- Find missing dimensions of solids (Block 9 &10)
- Graph exponential functions (Block 11)
- Identify characteristics of exponential functions (Block 11)
- Identify functions in tabular and graphical form (Block 12)
- Identify intervals of functions that are increasing or decreasing (Block 12)
- Identify intervals of functions that are positive or negative $f(x) > 0$ or $f(x) < 0$ (Block 12)
- Identify 3D shapes and composites of shapes (Time permitting extension)
- How to optimize (minimize) surface area of a given volume (Time permitting extension)

Resources

Core Text: Gerver, Robert K., and Richard J. Sgroi. *Financial Algebra: Advanced Algebra with Financial Applications*. National Geographic Learning/Cengage, 2021.

Suggested Resources: Desmos, Geogebra, Albert.io

UNIT 4: Banking & Credit

Summary and Rationale

This unit focuses on risk and reward. The degree of risk and the degree of reward depends upon the venture undertaken as well as the investor's comfort level with monetary risks. Students will be asked to consider the perceived risks and rewards of savings accounts and checking accounts. Students will practice reconciling a bank statement and explore compound interest using the compound interest formula. Throughout this unit, students manipulate checking accounts and have the opportunity to present knowledge regarding healthy banking practices. Students will build upon prior knowledge and skills in arithmetic and mathematics as they apply these concepts in real contexts. Students will have the opportunity to evaluate how the fields of psychology, government, sociology, business, and humanities impact real world decision-making processes and how relevant quantitative and computational thinking is for long-term success in life.

Recommended Pacing

14-16 days

State Standards

Standard A-SSE

CPI #	Cumulative Progress Indicator (CPI)
1	Interpret expressions that represent a quantity in terms of its context.
3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
3c	Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15t$ can be rewritten as $(1.151/12)12t \approx 1.01212t$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Standard A-CED Creating Equations

CPI #	Cumulative Progress Indicator (CPI)
4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.

Standard F-IF

CPI #	Cumulative Progress Indicator (CPI)
1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F
7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8b	Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t , y = (0.97)t , y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.

Standard F-BF Building Functions

CPI #	Cumulative Progress Indicator (CPI)
1	Write a function that describes a relationship between two quantities

Instructional Focus

Unit Enduring Understandings

- Strategies for maintaining a checking account promote wealth
- Compound interest impacts both the bank and the banker
- Budgets create boundaries and develop healthy disciplines for life-long spending and wealth
- management
- Energy and service providers are a cornerstone in society and literacy is an acquired skill to understand fee structures
- Employment opportunities will impact budget considerations

Unit Essential Questions

- How does one maintain a checking account?
- How does compound interest benefit the bank and the banker?
- How does a budget establish healthy spending habits?
- What are need expenses and want expenses?
- Why are employment options/ income options necessary in light of budgetary considerations?
- How is the price and cost of a product different and why is that important?

Objectives

Students will know:
- Checking accounts and check register
- Bank statements with spreadsheets
- Savings account overview
- simple interest
- Compound interest/ Compound interest formula
- Continuously compounded interest
- Future values of an investment with their graphs

Vocabulary: Checking account, check, EFT, payee, check clearing, ATM, canceled, insufficient funds, interest, single account, joint account, debits, credits, statement period, starting and ending balance, outstanding deposits, outstanding checks, balancing, reconciling, interest rate, APR, simple interest formula, maturity, arithmetic sequence, finite, infinite, compound interest, crediting, limits, function, rational function, continuous compounding, exponential base (e), continuous compound interest formula, inflation, ACH transactions

Students will be able to:

- Complete a checking account spreadsheet. (Section 2.1 &Section 2.3 Combined) (Block 1)
- Reconcile a checking account with a bank statement using a spreadsheet (Section 2.1 &Section 2.3 Combined) (Block 1)

- Define the basic vocabulary of savings accounts (Section 2.1 &Section 2.3 Combined) (Block 1)
- compute simple interest using the simple interest formula (Section 2.4) (Block 2)
- Explain the concept of getting interest on your account (Section 2.4) (Block 2)
- Compute compound interest using the formula and spreadsheet (Section 2.5) (Block 3)
- Compute interest on an account that is continuously compounded (Section 2.6) (Block 4)
- Calculate the future value of a periodic deposit investment. (Section 2.7) (Block 5)
- Graph and interpret the future value function (Section 2.7) (Block 6)
- Calculate the present value of an investment Section 2.8) (Block 7)
- Calculate the future value of an investment Section 2.8) (Block 7)
- Construct exponential and logarithmic models of a situation (Section 2.9) (Blocks 8-10)
- Evaluate logarithms (Section 2.9) (Blocks 8-10)
- Express exponential models as logarithms (Section 2.9) (Blocks 8-10)
- Demonstrate and use the change of base formula (Section 2.9) (Blocks 8-10)
- Evaluate common logarithms and natural logarithms (Section 2.9) (Blocks 8-10)
- Compute monthly payments and interest on loans (Section 3.2) (Block 11)
- Explain the options available for student loans (Section 3.3) (Block 12)
- Calculate the interest due in various student loan situations and apply the simple interest formula (Section 3.3) (Block 12)
- Determine the interest on a loan given the principal, time and APR (Section 3.4) (Block 13)
- Use logarithmic loan length formula to determine the term of a loan (Section 3.4) (Block 13)
- Use regression to determine the curve of best fit using data from a loan table (Section 3.4) (Block 13)
- Define the basic vocabulary of credit cards and understand the entries in credit card statements (Section 3.5 and 3.6) (Block 14)
- Compute an average daily balance for a credit card (Section 3.5 and 3.6) (Block 14)

Resources

Core Text: Gerver, Robert K., and Richard J. Sgroi. *Financial Algebra: Advanced Algebra with Financial Applications*. National Geographic Learning/Cengage, 2021.

Suggested Resources: Desmos, Geogebra, Albert.io

UNIT 5: Algebra Extension 3: Polynomials

Summary and Rationale
This unit extends the understanding of linear functions to quadratic functions and higher-order polynomials and their transformations. In the context of these functions, students will explore end behavior, the relationship between roots and x-intercepts, the multiplicity of roots, and solving methods. Modeling and problem-solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.
Recommended Pacing
12-14 Blocks
State Standards

Standard A-CED Creating Equations

CPI #	Cumulative Progress Indicator (CPI)
1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*
2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Standard A-APR Arithmetic with Polynomials and Rational Expressions

CPI #	Cumulative Progress Indicator (CPI)
3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
4	Prove polynomial identities and use them to describe numerical relationships.

Standard A-REI Reasoning with Equations and Inequalities

CPI #	Cumulative Progress Indicator (CPI)
4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are quadratic functions.

Standard A-SSE Reasoning with Equations and Inequalities

CPI #	Cumulative Progress Indicator (CPI)
1	Interpret expressions that represent a quantity in terms of its context.
1a	Interpret parts of an expression, such as terms, factors, and coefficients.
2	Use the structure of an expression to identify ways to rewrite it.
3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
3a	Factor a quadratic expression to reveal the zeros of the function it defines.

3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Standard F-IF Reasoning with Equations and Inequalities

CPI #	Cumulative Progress Indicator (CPI)
2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
4	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
5	Graph linear and quadratic functions and show intercepts, maxima, and minima.
8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Standard F-BF Building Functions

CPI #	Cumulative Progress Indicator (CPI)
1b	Combine standard function types using arithmetic operations.

Instructional Focus

Unit Enduring Understandings

- Roots and vertices of functions have real-world meaning.
- Polynomial shapes can be predicted.
- Graphs allow you to estimate solutions.
- There are mathematical patterns everywhere.
- To change one parameter of a function will change the look of the image.
- The zeroes of a function are the roots or x-intercepts

Unit Essential Questions

- What type of equation would model specific real-world data?
- Why is the vertex an important point of a parabola?
- How can factoring help graph a polynomial function?
- Can you find the roots of a graph without factoring?
- What are the critical attributes of a given function?
- What will you expect a family of functions to look like?
- How are the x-intercepts of a quadratic function related to the solutions of a quadratic equation?
- How many ways can you find the roots of a quadratic function?

Objectives

Students will know:

- How to write an equation of a function given critical attributes of the function.
- An algorithm for factoring quadratic functions.
- The quadratic formula can be derived by completing the square on a standard form quadratic equation
- A quadratic function can be written in vertex form, standard form, and intercept form.
- How to find critical attributes of a parabola given a specific form of a quadratic function.
- The y-intercept occurs when
- $x = 0$ and the x-intercept occurs when $y = 0$.
- The roots (solutions) to a quadratic equation are the zeros (x-intercepts) of the related quadratic function.
- Odd degree polynomial functions have the different end behavior moving towards
- $+ \infty / - \infty$

- Even degree polynomial functions have the same end behavior moving towards $+\infty / -\infty$
- How to express domain and range for polynomial functions using appropriate notation
- How to express end-behavior using proper notation
- The general shapes of several polynomial functions
- The factors of polynomial equations (roots) connect to the x-intercepts (zeros) of a polynomial function.

Vocabulary: polynomial, combine like terms, degree, leading coefficient, quadratic function, parabola, vertex form, intercept form, standard form, factoring by grouping, difference of squares, perfect square trinomials, roots, zeroes, solutions, end behavior, multiplicity, x-intercepts, y-intercepts, critical attributes

Students will be able to:
- Perform operations with polynomials(Addition, subtraction, multiplication) (Block 1)
- Factor to find roots of quadratic functions including difference of squares and perfect square trinomials (Block 2 & 3)
- Solve the same quadratic equation four ways: factoring, quadratic formula, completing the square, looking at the related graph (Block 4&5)
- Identify critical attributes of a graph from the three forms of a quadratic function (Block 5)
- Find x- and y-intercepts of graphs (Block 6&7)
- Graph analysis of quadratic functions (Block 6 & 7)
- Identify critical attributes of a graph from the three forms of a quadratic function (Block 6 & 7)
- Use various quadratic factoring techniques on higher order polynomials to find zeros – i.e.: factor by grouping and factor using u-substitution (Block 8)
- Identify end behaviors of a polynomial function (odd vs. even degree) and the behavior at any x-intercept (multiplicity of roots) (Block 9)
- Connect x-intercepts (zeros) of functions to factors of equations (roots) (Block 9)
- Graph polynomial functions using critical attributes and multiplicity (Block 10)
- Write an equation of a polynomial functions given some or all of its roots (Block 11)
- Write an equation for a given sketch of a graph (Block 11)
- Find the zeros and sketch a polynomial function given the linear factors of a polynomial expression (Block 12)

Resources

Core Text: Gerver, Robert K., and Richard J. Sgroi. *Financial Algebra: Advanced Algebra with Financial Applications*. National Geographic Learning/Cengage, 2021.

Suggested Resources: Desmos, Geogebra, Albert.io

UNIT 6: Automobile Unit

Summary and Rationale	

The automobile unit will take a look at multiple mathematical concepts through the lens of automobiles. Students will calculate sales tax and advertisements for cars, analyze measures of central tendency, find probabilities and conditional probabilities, determine the appreciation or depreciation of a car's value, and analyze accident data. A foundational skill for this unit will be properties of logarithms and solving logarithms. This will be introduced as a traditional topic before the Historical and Exponential Depreciation lesson

Recommended Pacing	

14-16 Blocks

State Standards	

Standard F-IF Interpreting Functions

CPI #	Cumulative Progress Indicator (CPI)
1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Standard F-LE Interpreting Functions

CPI #	Cumulative Progress Indicator (CPI)
2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standard A-CED Creating Equations

CPI #	Cumulative Progress Indicator (CPI)
1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions
4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Standard S-ID Interpreting Categorical and Quantitative Data

CPI #	Cumulative Progress Indicator (CPI)
1	Represent data with plots on the real number line (dot plots, histograms, and box plots).
2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Standard S-CP Conditional Probability and the Rules of Probability

CPI #	Cumulative Progress Indicator (CPI)
1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results*
5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Standard F-BF Building Functions

CPI #	Cumulative Progress Indicator (CPI)
5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

Standard S-MD Using Probability to Make Decisions

CPI #	Cumulative Progress Indicator (CPI)
1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

Instructional Focus

Unit Enduring Understandings

- Consumers should determine their needs and research options prior to purchasing an automobile.
- Financing options are available for purchasing an automobile.

Unit Essential Questions

- How do consumers prepare for purchasing an automobile?
- What are the various financing options available to consumers?

Objectives

Students will know:
- The cost of classified ads for used cars
- The cost of sales tax on automobiles
- Quartiles and interquartile ranges
- Frequency distributions
- Box-and-whisker plots and stem and leaf plots
- Types of auto insurance coverage, claim costs, and policy costs
- two-way tables
- Conditional probabilities
- independent and dependent events
- Venn diagrams

- Depreciation equations
- Equivalent expressions and equations
- Quadratic equations
- A logarithm can be used to find the value of an exponent to a power function.
- Logarithmic functions and exponential functions are inverses of each other.
- Basic properties of logs:

$$\log \log 1 \ = \ 0; \ \ln \ln 1 \ = \ 0$$
$$b \ = \ 1; \ \ln \ln e \ = \ 1$$
$$\log \log a^{x} = \ a \ ; \ \ln \ln e^{x} \ = \ x$$

Vocabulary: sales tax, domain, piecewise function, split function, cusp, data, measures of central tendency, quartiles, lower quartile, upper quartile, interquartile range, stem-and-leaf plot, box-and-whisker plot, box plot, modified box plot, liable, negligent, automobile insurance, premium, claim, liability insurance, no-fault insurance, comprehensive insurance, collision insurance, car-rental insurance, surcharge, deductible, depreciate, appreciate, expense functions, fixed expense, variable expense, reaction time, braking distance

Students will be able to:
- Calculate sales tax or discounts on an item (Block 1)
- Analyze the cost using a piecewise function (Block 1)
- Calculate measures of central tendencies (Block 2)
- Compute quartiles and interquartile ranges, and outliers (Block 2)
- Use box-and-whisker plots to analyze data sets (Block 3)
- Use stem-and-leaf plots to analyze data sets (Block 3)
- Determine the amount owed by an insurance company vs. an individual for premiums, deductibles, and claims. (Block 4)
- Calculate a probability by reading and interpreting a two-way frequency table. (Block 5)
- Identify if an event is independent or associated based on a two-way frequency table. (Block 5)
- Create and interpret Venn Diagrams. (Block 6)
- Use data sets to calculate conditional probabilities. (Block 6)
- Write, interpret, and graph straight-line appreciation equations. (Block 7)
- Write, interpret, and graph exponential appreciation/depreciation equations.(Block 7)
- Solve the equation y=A(1r)x or y=P(1r)t for any of the given variables. (Block 10)
- Evaluate a logarithmic expression. (Block 10)
- Rewrite an equation from exponential form to logarithmic form and vice versa (Block 10)
- Solve logarithmic equations using basic properties of logs. (Block 11)
- Solve exponential equations for the exponent using properties of logarithms(Block 12)
- Use formulas to calculate reaction time and distance (Block 13)
- Solve for the length of a skid mark or how long it will take for a car to stop in an investigation. (Block 13 &14)

Resources

Core Text: Gerver, Robert K., and Richard J. Sgroi. *Financial Algebra: Advanced Algebra with Financial Applications*. National Geographic Learning/Cengage, 2021.
Suggested Resources: Desmos, Geogebra, Albert.io, Citizen Math

UNIT 7: Algebra Extension 4: Trigonometry

Summary and Rationale
This unit extends students' understanding of the basic trigonometric functions used with triangles (sine, cosine, and tangent) to explore their operation on a coordinate plane. Students will first explore the unit circle, reference angles, and coterminal angles, and will then transfer that understanding to build functions on the coordinate plane. Graph analysis of amplitude, period, and phase shift will be explored through an examination of the transformations on these functions, and a discussion of domain and range will be embedded throughout. Modeling and problem-solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

Recommended Pacing
9-10 Blocks

State Standards

Standard F-TF Trigonometric Functions

CPI #	Cumulative Progress Indicator (CPI)
1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3	(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
4	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
8	Prove the Pythagorean identity $sin^2(\Theta) + cos(\Theta) = 1$ and use it to and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.

Standard G-SRT Similarity, Right Triangles, and Trigonometry

CPI #	Cumulative Progress Indicator (CPI)
1	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Instructional Focus

Unit Enduring Understandings

- Analyzing trends predict behavior.
- There are many ways to solve a problem.
- Relationships vary by situation.
- Changing one or many pieces can change the whole.
- Characteristics allow you to make predictions.
- Multiple models exist for a single scenario.
- Procedures can be reversed.
- There are six trigonometric ratio functions related to a right triangle.

	•	Trig functions allow you to solve any right triangle.		

Unit Essential Questions

- Why are predictions beneficial?
- When there are multiple approaches, how should you choose the best method?
- Can understanding a relationship help make a prediction?
- How do the parts affect the whole?
- How do your characteristics relate you to your family?
- Is there a need for different viewpoints?
- What is the effect of interchanging parts?
- How can you use trig ratios to solve problems that model triangles?
- How do you evaluate the six trig functions for any given reference angle measure without a calculator?

Objectives

Students will know:

- There are six trigonometric ratio functions related to a right triangle
- The radian measure of an angle is the length of the arc on the unit circle
- The equation of the unit circle
- On the unit circle,
- $x = cos\ (\theta)$ and $y = \sin\ (\theta)$
- Real world phenomena can be modeled using trig functions
- The Pythagorean identity $sin^2(\Theta) + cos(\Theta) = 1$
- The measure of an angle in degrees or radians is positive if its rotation is counterclockwise from the positive x-axis.
- End behavior of oscillating graphs is undefined since periodic functions will not converge to a single value
- Proper notation for domain and range of trigonometric functions
- One complete rotation is 360 degrees or 2pi radians
- The value of a trigonometric ratio is dependent on what quadrant the angle's terminal side is in
- The period does not influence range but amplitude and vertical shift do

Vocabulary: six trigonometric functions: sine, cosine, tangent, secant, cosecant, cotangent, reference angle, quadrantal angles, standard position, x axis, positive angle (counterclockwise), negative angle (clockwise), coterminal angles, asymptote, oscillating curve, period, amplitude, Pythagorean theorem, 30-60-90 and 45-45-90 special right triangles

Students will be able to:

- Use special right triangles to find missing lengths. (Block 1)
- Use the six trig functions to solve right triangles (Block 2)
- Evaluate six trig functions for any given angle measure without a calculator (Unit Circle Activity Block 3)
- Find a reference angle and a coterminal angle and know how they are related (Block 4)
- Given the value of one trigonometric function in a specific quadrant, find the value of the other five (Block 5)
- Convert between radians and degrees (Block 6)
- Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π /4 and π /6 (Block 7)
- Graph the parent functions $f(x) = sin\ (x)$, $f(x) = cos\ (x)$ and $f(x) = tan\ (x)$ (Block 8 & 9)
- Compare $f(x)$ to $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for trigonometric functions (Block 8 & 9)
- Identify the amplitude, period, phase shift, and vertical shift for sine and cosine (Block 8 & 9)
- Examine domain and range of all functions covered (Block 8 & 9)

Resources

Core Text: Gerver, Robert K., and Richard J. Sgroi. *Financial Algebra: Advanced Algebra with Financial Applications*. National Geographic Learning/Cengage, 2021.

Suggested Resources: Desmos, Geogebra, Albert.io