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AP Precalculus

Content Area:	Mathematics
Grade Span:	11-12
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COURSE OVERVIEW

Description

AP Precalculus prepares students for college-level mathematics and science courses through the study of four units: polynomial and rational functions; exponential and logarithmic functions; trigonometric and polar functions; and functions involving parameters, vectors, and matrices. Through regular practice, students build deep mastery of modeling and functions, and they examine scenarios through multiple representations. The course framework delineates content and skills common to college precalculus courses that are foundational for careers in mathematics, physics, biology, health science, social science, and data science.

Goals

In addition to the content standards, skills, and concepts set forth, this course also seeks to meet the Standards for Mathematical Practice. These practices include generally applied best practices for learning mathematics, such as understanding the nature of proof and having a productive disposition towards the subject, and are not tied to a particular set of content. These skills are applicable beyond a student's study of mathematics.
The eight Standards for Mathematical Practice are outlined below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Scope and Sequence

Unit	Topic	Length (90 Blocks)
1a	Rates of Change	5
1b	Characteristics of Polynomial and Rational Functions	8
1c	Transformations and Applications of Polynomial and Rational Functions	7
2a	Sequences, Data Modeling, and Exponential Functions	8
2b	Compositions, Inverses, and Logarithms	12
3a	Sine, Cosine, and Tangent: Functions, Graphs, and Transformations	9
3b	Inverse Trigonometry; Trigonometric Equations; and Cosecant, Secant, and Cotangent	6
3c	Polar Coordinates and Polar Functions	5
N/A	Cumulative Review for AP Exam (Above Units Only)	10
4a	Parametrics and Conics	10
4b	Vectors and Matrices	10

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, Numerical, Algebraic*. Pearson. **Suggested Resources:** AP Classroom, Desmos, Geogebra, Albert.io

UNIT 1a: Rates of Change

Summary and Rationale

In Unit 1, students develop an understanding of two key function concepts while exploring polynomial and rational functions. The first concept is covariation, or how output values change in tandem with changing input values. The second concept is rates of change, including the average rate of change, the rate of change at a point, and changing rates of change. The central idea of a function as a rule for relating two simultaneously changing sets of values provides students with a vital tool that has many applications, in nature, human society, and business and industry. For example, the idea of crop yield increasing but at a decreasing rate or the efficacy of medicine decreasing but at an increasing rate are important understandings that inform critical decisions.

Recommended Pacing

5 Blocks

AP Mathematical Practices

Practice 1 - Procedural and Symbolic Fluency

1.A	Solve equations and inequalities represented analytically, with and without technology.
1.B	Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C	Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.

Practice 2 - Multiple Representations

2.A	Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B	Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.

Practice 3 - Communication and Reasoning

3.A	Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools
3.B	Apply numerical results in a given mathematical or applied context.
3.C	Support conclusions or choices with a logical rationale or appropriate data.

Instructional Focus

Unit Enduring Understandings

- Compare two quantities and see how they vary with each other such as directly or indirectly
- Interpret graphs to study positive, negative and constant rate of change.
- Knowing rate of change can help find instantaneous rate of change e.g. how speed varies with time.

Unit Essential Questions

- How do we model the intensity of light from its source?
- How can I use data and graphs to figure out the best time to purchase event tickets?
- How can we adjust known projectile motion models to account for changes in conditions?

Objectives

Students will know:

- A function is a mathematical relation that maps a set of input values to a set of output values such that each input value is mapped to exactly one output value. The set of input values is called the domain of the function, and the set of output values is called the range of the function. The variable representing input values is called the independent variable, and the variable representing output values is called the dependent variable.

- The input and output values of a function vary in tandem according to the function rule, which can be expressed graphically, numerically, analytically, or verbally.
- A function is increasing over an interval of its domain if, as the input values increase, the output values always increase. That is, for all a and b in the interval, if a < b, then f (a) < f (b)
- A function is decreasing over an interval of its domain if, as the input values increase, the output values always decrease. That is, for all a and b in the interval, if a < b, then f(a) > f (b).
- The graph of a function displays a set of input-output pairs and shows how the values of the function's input and output values vary
- The graph of a function displays a set of input-output pairs and shows how the values of the function's input and output values vary
- A verbal description of the way aspects of phenomena change together can be the basis for constructing a graph
- The graph of a function is concave up on intervals in which the rate of change is increasing.
- The graph of a function is concave down on intervals in which the rate of change is decreasing.
- The graph intersects the x-axis when the output value is zero. The corresponding input values are said to be zeros of the function.
- The average rate of change of a function over an interval of the function's domain is the constant rate of change that yields the same change in the output values as the function yielded on that interval of the function's domain. It is the ratio of the change in the output values to the change in input values over that interval
- The rate of change of a function at a point quantifies the rate at which output values would change were the input values to change at that point. The rate of change at a point can be approximated by the average rates of change of the function over small intervals containing the point, if such values exist.
- The rates of change at two points can be compared using average rate of change approximations over sufficiently small intervals containing each point, if such values exist.
- Rates of change quantify how two quantities vary together.
- A positive rate of change indicates that as one quantity increases or decreases, the other quantity does the same.
- A negative rate of change indicates that as one quantity increases, the other decreases.
- For a linear function, the average rate of change over any length input-value interval is constant.
- For a quadratic function, the average rates of change over consecutive equal-length input value intervals can be given by a linear function.
- The average rate of change over the closed interval [a, b] is the slope of the secant line from the point
- (a, f(a)) to (b, f(b)).
- Where a polynomial function switches between increasing and decreasing, or at the included endpoint of a polynomial with a restricted domain, the polynomial function will have a local, or relative, maximum or minimum output value. Of all local maxima, the greatest is called the global, or absolute, maximum. Likewise, the least of all local minima is called the global, or absolute, minimum.
- Between every two distinct real zeros of a nonconstant polynomial function, there must be at least one input value corresponding to a local maximum or local minimum.
- Polynomial functions of an even degree will have either a global maximum or a global minimum.
- Points of inflection of a polynomial function occur at input values where the rate of change of the function changes from increasing to decreasing or from decreasing to increasing. This occurs where the graph of a polynomial function changes from concave up to concave down or from concave down to concave up.

Vocabulary: function, domain, range, independent variable, dependent variable, function rule, concave up, concave down, zeros of a function, maxima, minima, global maxima, global minima

Students will be able to:

- Describe how the input and output values of a function vary together by comparing function values.
- Construct a graph representing two quantities that vary with respect to each other in a contextual scenario.
- Compare the rates of change at two points using average rates of change near the points.
- Describe how two quantities vary together at different points and over different intervals of a function.
- Determine the average rates of change for sequences and functions, including linear, quadratic, and other function types.
- Determine the change in the average rates of change for linear, quadratic, and other function types.
- Identify key characteristics of polynomial functions related to rates of change.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, Numerical, Algebraic*. Pearson.
Suggested Resources: AP Classroom (1.1-1.4), Desmos, Geogebra, Albert.io

UNIT 1b: Characteristics of Polynomial and Rational Functions

Summary and Rationale

In Unit 1, students develop understanding of two key function concepts while exploring polynomial and rational functions. The first concept is covariation, or how output values change in tandem with changing input values. The second concept is rates of change, including average rate of change, rate of change at a point, and changing rates of change. The central idea of a function as a rule for relating two simultaneously changing sets of values provides students with a vital tool that has many applications, in nature, human society, and business and industry. For example, the idea of crop yield increasing but at a decreasing rate or the efficacy of a medicine decreasing but at an increasing rate are important understandings that inform critical decisions.

Recommended Pacing

8 Blocks

AP Mathematical Practices

Practice 1 - Procedural and Symbolic Fluency

1.A	Solve equations and inequalities represented analytically, with and without technology.
1.B	Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C	Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.

Practice 2 - Multiple Representations

2.A	Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B	Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.

Practice 3 - Communication and Reasoning

3.A	Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools
3.B	Apply numerical results in a given mathematical or applied context.
3.C	Support conclusions or choices with a logical rationale or appropriate data.

Instructional Focus

Unit Enduring Understandings

- Order matters when composing functions.
- Basic rules of equality are the key to ensure accuracy in mathematics.
- Rational functions are not continuous because they have asymptotes.
- Polynomials have smooth turns and no sharp edges.
- A polynomial of degree n has precisely n linear factors in the complex number system.

Unit Essential Questions

- How do we model the intensity of light from its source?
- How can I use data and graphs to figure out the best time to purchase event tickets?
- How can we adjust known projectile motion models to account for changes in conditions?

Objectives

Students will know:

- If a is a complex number and p(a) = 0, then a is called a zero of the polynomial function p, or a root of p(x) = 0. If a is a real number, then (x − a) is a linear factor of p if and only if a is a zero of p

- If a linear factor (x - a) is repeated n times, the corresponding zero of the polynomial function has a multiplicity. A polynomial function of degree n has exactly n complex zeros when counting multiplicities.
- If a is a real zero of a polynomial function p, then the graph of y = p(x) has an x-intercept at the point (a, 0). Consequently, real zeros of a polynomial can be endpoints for intervals satisfying polynomial inequalities
- If a + bi is a non-real zero of a polynomial function p, then its conjugate a − bi is also a zero of p.
- If the real zero, a, of a polynomial function has even multiplicity, then the signs of the output values are the same for input values near x = a. For these polynomial functions, the graph will be tangent to the x-axis at x = a.
- The degree of a polynomial function can be found by examining the successive differences of the output values over equal-interval input values. The degree of the polynomial function is equal to the least value n for which the successive nth differences are constant.
- An even function is graphically symmetric over the line x = 0 and analytically has the property f (−x) = f (x)
- If n is even, then a polynomial of the form $p(x) = a_n x^n$, where n ≥ 1 and $a_n \neq 0$, is an even function.
- An odd function is graphically symmetric about the point (0,0) and analytically has the property f (−x) = − f (x). If n is odd, then a polynomial of the form $p(x) = a_n x^n$, where n ≥ 1 and $a_n \neq 0$, is an odd function.
- As input values of a nonconstant polynomial function increase without bound, the output values will either increase or decrease without bound
- As input values of a nonconstant polynomial function decrease without bound, the output values will either increase or decrease without bound
- The degree and sign of the leading term of a polynomial determines the end behavior of the polynomial function, because as the input values increase or decrease without bound, the values of the leading term dominate the values of all lower-degree terms.
- A rational function is analytically represented as a quotient of two polynomial functions and gives a measure of the relative size of the polynomial function in the numerator compared to the polynomial function in the denominator for each value in the rational function's domain.
- The end behavior of a rational function will be affected most by the polynomial with the greater degree, as its values will dominate the values of the rational function for input values of large magnitude. For input values of large magnitude, a polynomial is dominated by its leading term. Therefore, the end behavior of a rational function can be understood by examining the corresponding quotient of the leading terms.
- If the polynomial in the numerator dominates the polynomial in the denominator for input values of large magnitude, then the quotient of the leading terms is a nonconstant polynomial, and the original rational function has the end behavior of that polynomial. If that polynomial is linear, then the graph of the rational function has a slant asymptote parallel to the graph of the line.
- If neither polynomial in a rational function dominates the other for input values of large magnitude, then the quotient of the leading terms is a constant, and that constant indicates the location of a horizontal asymptote of the graph of the original rational function.
- If the polynomial in the denominator dominates the polynomial in the numerator for input values of large magnitude, then the quotient of the leading terms is a rational function with a constant in the numerator and nonconstant polynomial in the denominator, and the graph of the original rational function has a horizontal asymptote at y = 0.
- When the graph of a rational function r has a horizontal asymptote at y = b, where b is a constant, the output values of the rational function get arbitrarily close to b and stay arbitrarily close to b as input values increase or decrease without bound.
- The real zeros of a rational function correspond to the real zeros of the numerator for such values in its domain
- The real zeros of both polynomial functions of a rational function r are endpoints or asymptotes for intervals satisfying the rational function inequalities r (x) ≥ 0 or r (x) ≤ 0.
- If the value a is a real zero of the polynomial function in the denominator of a rational function and is not also a real zero of the polynomial function in the numerator, then the graph of the rational function has a vertical asymptote at x = a. Furthermore, a vertical asymptote also occurs at x = a if the multiplicity of a as a real zero in the denominator is greater than its multiplicity as a real zero in the numerator.

- Near a vertical asymptote, x = a, of a rational function, the values of the polynomial function in the denominator are arbitrarily close to zero, so the values of the rational function r increase or decrease without bound.
- If the multiplicity of a real zero in the numerator is greater than or equal to its multiplicity in the denominator, then the graph of the rational function has a hole at the corresponding input value.
- If the graph of a rational function r has a hole at x = c, then the location of the hole can be determined by examining the output values corresponding to input values sufficiently close to c. If input values sufficiently close to c correspond to output values arbitrarily close to L, then the hole is located at the point with coordinates (c, L).

Vocabulary: Asymptotes, Polynomials, Rationals, Even, Odd, Zeros, factors, complex, imaginary, holes, degree, leading coefficient, end behavior, multiplicity.

Students will be able to:
- Identify key characteristics of a polynomial function related to its zeros when suitable factorizations are available or with technology.
- Determine if a polynomial function is even or odd.
- Describe end behaviors of polynomial functions.
- Describe end behaviors of rational functions.
- Determine the zeros of rational functions.
- Determine vertical asymptotes of graphs of rational functions.
- Determine holes in graphs of rational functions.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, Numerical, Algebraic*. Pearson.
Suggested Resources: AP Classroom (1.5-1.10), Desmos, Geogebra, Albert.io

UNIT 1c: Transformations and Applications of Polynomial and Rational Functions

Summary and Rationale

In Unit 1, students develop an understanding of two key function concepts while exploring polynomial and rational functions. The first concept is covariation, or how output values change in tandem with changing input values. The second concept is rates of change, including average rate of change, rate of change at a point, and changing rates of change. The central idea of a function as a rule for relating two simultaneously changing sets of values provides students with a vital tool that has many applications, in nature, human society, and business and industry. For example, the idea of crop yield increasing but at a decreasing rate or the efficacy of medicine decreasing but at an increasing rate are important understandings that inform critical decisions.

Recommended Pacing

7 Blocks

AP Mathematical Practices

Practice 1 - Procedural and Symbolic Fluency

1.A	Solve equations and inequalities represented analytically, with and without technology.
1.B	Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C	Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.

Practice 2 - Multiple Representations

2.A	Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B	Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.

Practice 3 - Communication and Reasoning

3.A	Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools
3.B	Apply numerical results in a given mathematical or applied context.
3.C	Support conclusions or choices with a logical rationale or appropriate data.

Instructional Focus

Unit Enduring Understandings

- A polynomial can be written as a product of linear factors.
- Pascal's Triangle can be used to find n products of a binomial and written as a polynomial of degree n.
- A polynomial of degree n has precisely n linear factors in the complex number system.
- A polynomial with even number of repeated zeros bounces off at that zero and with odd number of repeated zeros crosses the x-axis

Unit Essential Questions

- How do we model the intensity of light from its source?
- How can I use data and graphs to figure out the best time to purchase event tickets?
- How can we adjust known projectile motion models to account for changes in conditions?

Objectives

Students will know:

- Because the factored form of a polynomial or rational function readily provides information about real zeros, it can reveal information about x-intercepts, asymptotes, holes, domain, and range.
- The standard form of a polynomial or rational function can reveal information about end behaviors of the function.
- The information extracted from different analytic representations of the same polynomial or rational function can be used to answer questions in context.
- Polynomial long division is an algebraic process similar to numerical long division involving a quotient and remainder. If the polynomial f is divided by the polynomial g , then f can be rewritten as f (x) = g (x)q (x) + r (x), where q is the quotient, r is the remainder, and the degree of r is less than the degree of g
- Polynomial long division is an algebraic process similar to numerical long division involving a quotient and remainder. If the polynomial f is divided by the polynomial g , then f can be rewritten as f (x) = g (x)q (x) + r (x), where q is the quotient, r is the remainder, and the degree of r is less than the degree of g
- The binomial theorem utilizes the entries in a single row of Pascal's Triangle to more easily expand expressions of the form $(a + b)^n$, including polynomial functions of the form $p(x) = (x + c)^n$, where c is a constant.
- The function g (x) = f (x)+k is an additive transformation of the function f that results in a vertical translation of the graph of f by k units.
- The function g (x) = f (x+h) is an additive transformation of the function f that results in a horizontal translation of the graph of f by −h units.
- The function g (x) = a f(x), where a ≠ 0, is a multiplicative transformation of the function f that results in a vertical dilation of the graph of f by a factor of a . If a < 0, the transformation involves a reflection over the x-axis.
- The function g (x) = f (bx), where b ≠ 0, is a multiplicative transformation of the function f that results in a horizontal dilation of the graph of f by a factor of $|\frac{1}{b}|$. If b < 0, the transformation involves a reflection over the y-axis.
- Additive and multiplicative transformations can be combined, resulting in combinations of horizontal and vertical translations and dilations.
- The domain and range of a function that is a transformation of a parent function may be different from those of the parent function.
- Linear functions model data sets or aspects of contextual scenarios that demonstrate roughly constant rates of change.
- Quadratic functions model data sets or aspects of contextual scenarios that demonstrate roughly linear rates of change, or data sets that are roughly symmetric with a unique maximum or minimum value.
- Geometric contexts involving area or two dimensions can often be modeled by quadratic functions. Geometric contexts involving volume or three dimensions can often be modeled by cubic functions.
- Polynomial functions model data sets or contextual scenarios with multiple real zeros or multiple maxima or minima.
- A polynomial function of degree n models data sets or contextual scenarios that demonstrate roughly constant nonzero nth differences.
- A polynomial function of degree n or less can be used to model a graph of n + 1 points with distinct input values
- A piecewise-defined function consists of a set of functions defined over nonoverlapping domain intervals and is useful for modeling a data set or contextual scenario that demonstrates different characteristics over different intervals.
- A model may have underlying assumptions about what is consistent in the model.
- A model may have underlying assumptions about how quantities change together.
- A model may require domain restrictions based on mathematical clues, contextual clues, or extreme values in the data set.
- A model may require range restrictions, such as rounding values, based on mathematical clues, contextual clues, or extreme values in the data set
- A model can be constructed based on restrictions identified in a mathematical or contextual scenario.

- A model of a data set or a contextual scenario can be constructed using transformations of the parent function.
- A model of a data set can be constructed using technology and regressions, including linear, quadratic, cubic, and quartic regressions.
- A piecewise-defined function model can be constructed through a combination of modeling techniques.
- Construct a rational function model based on a context. Data sets and aspects of contextual scenarios involving quantities that are inversely proportional can often be modeled by rational functions. For example, the magnitudes of both gravitational force and apply a function model to answer questions about a data set or contextual scenario.
- A model can be used to draw conclusions about the modeled data set or contextual scenario, including answering key questions and predicting values, rates of change, average rates of change, and changing rates of change. Appropriate units of measure should be extracted or inferred from the given context.

Vocabulary: Asymptotes, Polynomials, Rationals, Even, Odd, Zeros, factors, complex, imaginary, holes, degree, leading coefficient, end behavior, multiplicity, binomial theorem, Pascal's Triangle, quotient.

Students will be able to:
- Rewrite polynomial and rational expressions in equivalent forms
- Determine the quotient of two polynomial functions using long division.
- Rewrite the repeated product of binomials using the binomial theorem.
- Construct a function that is an additive and/or multiplicative transformation of another function.
- Identify an appropriate function type to construct a function model for a given scenario.
- Describe assumptions and restrictions related to building a function model.
- Construct a linear, quadratic, cubic, quartic, polynomial of degree n, or related piecewise-defined function model.
- Construct a rational function model based on a context.
- Apply a function model to answer questions about a data set or contextual scenario.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, Numerical, Algebraic*. Pearson.
Suggested Resources: AP Classroom (1.11-1.14), Desmos, Geogebra, Albert.io

UNIT 2a: Sequences, Data Modeling, and Exponential Functions

Summary and Rationale

In Unit 2, students build an understanding of exponential and logarithmic functions. Exponential and logarithmic function models are widespread in the natural and social sciences. When an aspect of a phenomenon changes proportionally to the existing amount, exponential and logarithmic models are employed to harness the information. Exponential functions are key to modeling population growth, radioactive decay, interest rates, and the amount of medication in a patient. Logarithmic functions are useful in modeling sound intensity and frequency, the magnitude of earthquakes, the pH scale in chemistry, and the working memory in humans. The study of these two function types touches careers in business, medicine, chemistry, physics, education, and human geography, among others.

Recommended Pacing

8 Blocks

AP Mathematical Practices

Practice 1 - Procedural and Symbolic Fluency

1.A	Solve equations and inequalities represented analytically, with and without technology.
1.B	Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C	Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.

Practice 2 - Multiple Representations

2.A	Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B	Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.

Practice 3 - Communication and Reasoning

3.A	Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools
3.B	Apply numerical results in a given mathematical or applied context.
3.C	Support conclusions or choices with a logical rationale or appropriate data.

Instructional Focus

Unit Enduring Understandings

- Analyzing trends predict behavior.
- Relationships vary by situation.
- Changing one or many pieces can change the whole.
- Procedures can be reversed.
- Logarithmic functions and exponential functions are inverses of one another.
- Appreciation and depreciation affect the rate of growth.

Unit Essential Questions

- How can I make a single model that merges the interest I earn from my bank with the taxes that are due, so I can know how much I will have in the end?
- How can we adjust the scale of distance for a model of planets in the solar system, so the relationships among the planets are easier to see?
- If different functions can be used to model data, how do we pick which one is best?

Objectives

Students will know:

- A sequence is a function from the whole numbers to the real numbers. Consequently, the graph of a sequence consists of discrete points instead of a curve.
- Successive terms in an arithmetic sequence have a common difference, or constant rate of change.
- The general term of an arithmetic sequence with a common difference d is denoted by an and is given by an = a0 + dn, where a0 is the initial value, or by an = ak + d(n − k), where ak is the kth term of the sequence.
- Successive terms in a geometric sequence have a common ratio, or constant proportional change.
- The general term of a geometric sequence with a common ratio r is denoted by gn and is given by the equation gn = gkr(n-k), where gk is the kth term of the sequence
- Increasing arithmetic sequences increase equally with each step, whereas increasing geometric sequences increase by a larger amount with each successive step.
- Linear functions of the form f(x) = mx + b are similar to arithmetic sequences of the form an = a0 + dn, as both are an initial value (y-intercept) plus repeated addition of a constant rate of change (slope).
- Similar to arithmetic sequences of the form an = ak + d(n-k), which are based on a known difference (d) and a kth term, linear functions can be expressed in the form f(x) = yi + m(x-xi), which are based on a slope (m) and a point (xi, yi).
- Exponential functions of the form f(x) = abx are similar to geometric sequences of the form gn = g0rn, as both are an initial value times repeated multiplication by a constant proportion.
- Similar to geometric sequences of the form gn = gkr(n-k), which are based on a known ratio (r) and a kth term, exponential functions can be expressed in the form f(x) = yir(x-xi) based on a ratio (r) and a point (xi, yi).
- Sequences and their corresponding functions may have different domains.
- Over equal-length input-value intervals, if the output values of a function change at constant rate, then the function is linear; if the output values of a function change proportionally, then the function is exponential.
- Linear functions of the form f(x) = mx + b and exponential functions of the form f(x) = abx can both be expressed analytically in terms of an initial value and a constant involved with change. However, linear functions are based on addition, while exponential functions are based on multiplication.
- Arithmetic sequences, linear functions, geometric sequences, and exponential functions all have the property that they can be determined by two distinct sequence or function values.
- The general form of an exponential function is f(x) = abx, with the initial value a, where a =/= 0, and the base b, where b > 0, and b =/= 1. When a > 0 and b > 1, the exponential function demonstrates exponential growth. When a > 0 and 0 < b < 1, the exponential function demonstrates exponential decay.
- When the natural numbers are input values in an exponential function, the input value specifies the number of factors of the base to be applied to the function's initial value. The domain of an exponential function is all real numbers.
- Because the output values of exponential functions in general form are proportional over equal-length input-value intervals, exponential functions are always increasing or always decreasing, and their graphs are always concave up or always concave down. Consequently, exponential functions do not have extrema except on a closed interval, and their graphs do not have points of inflection.
- If the values of the additive transformation function g(x) = f(x) + k of any function f(x) are proportional over equal-length input-value intervals, then f(x) is exponential.
- For an exponential function in general form, as the input values increase or decrease without bound, the output values will increase or decrease without bound or will get arbitrarily close to zero.
- The product property for exponents states that bmbn = bm+n.
- The power property for exponents states that (bm)n = bmn.
- The negative exponent property states that b-n = 1/bn.
- The value of an exponential expression involving an exponential fraction, such as b1/k, where k is a natural number, is the kth root of b.
- Exponential functions model growth patterns where successive output values over equal-length input-value intervals are proportional. When the input values are whole numbers, exponential functions model situations of repeated multiplication of a constant to an initial value.

- A constant may need to be added to the dependent variable values of a data set to reveal a proportional growth pattern.
- An exponential function model can be constructed from an appropriate ratio and initial value or from two input-output pairs. The initial value and the base can be found by solving a system of equations resulting from the two input-output pairs.
- Exponential function models can be constructed by applying transformations to f(x) = abx based on characteristics of a contextual scenario or data set.
- Exponential function models can be constructed for a data set with technology using exponential regressions.
- The natural base e, which is approximately 2.718, is often used as the base in exponential functions that model contextual scenarios.
- For an exponential model in general form f(x) = abx, the base of the exponent b can be understood as a growth factor in successive unit changes in the input values and is related to a percent change in context.
- Equivalent forms of an exponential function can reveal different properties of the function.
- Exponential models can be used to predict values for the dependent variable, depending on the contextual constraints on the domain.
- Two variables in a data set that demonstrate a slightly changing rate of change can be modeled by linear, quadratic, and exponential function models.
- Models can be compared based on contextual clues and applicability to determine which model is most appropriate.
- A model is justified as appropriate for a data set if the graph of the residuals of a regression, the residual plot, appear without pattern.
- The difference between the predicted and actual values is the error in the model. Depending on the data set and context, it may be more appropriate to have an underestimate or overestimate for any given interval.

Vocabulary: Exponential Growth, Exponential Decay, Compound interest, number e, term, compounding method, Annually, Quarterly, asymptote, logistic, intercept, initial, final, time, Principal, half life, radioactive, Newton's law, Logarithmic, inverse, ph, intensity, laws of log, natural log

Students will be able to:

- Express arithmetic sequences found in mathematical and contextual scenarios as functions of the whole numbers.
- Express geometric sequences found in mathematical and contextual scenarios as functions of the whole numbers.
- Construct functions of the real numbers that are comparable to arithmetic and geometric sequences.
- Describe similarities and differences between linear and exponential functions.
- Identify key characteristics of exponential functions.
- Rewrite exponential expressions in equivalent forms.
- Construct a model for situations involving proportional output values over equal-length input-value intervals.
- Apply exponential models to answer questions about a data set or contextual scenario.
- Construct linear, quadratic, and exponential models based on a data set.
- Validate a model constructed from a data set.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, Numerical, Algebraic*. Pearson.
Suggested Resources: AP Classroom (2.1-2.6), Desmos, Geogebra, Albert.io

UNIT 2b: Compositions, Inverses, and Logarithms

Summary and Rationale

In Unit 2, students build an understanding of exponential and logarithmic functions. Exponential and logarithmic function models are widespread in the natural and social sciences. When an aspect of a phenomenon changes proportionally to the existing amount, exponential and logarithmic models are employed to harness the information. Exponential functions are key to modeling population growth, radioactive decay, interest rates, and the amount of medication in a patient. Logarithmic functions are useful in modeling sound intensity and frequency, the magnitude of earthquakes, the pH scale in chemistry, and the working memory in humans. The study of these two function types touches careers in business, medicine, chemistry, physics, education, and human geography, among others.

Recommended Pacing

12 Blocks

AP Mathematical Practices

Practice 1 - Procedural and Symbolic Fluency

1.A	Solve equations and inequalities represented analytically, with and without technology.
1.B	Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C	Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.

Practice 2 - Multiple Representations

2.A	Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B	Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.

Practice 3 - Communication and Reasoning

3.A	Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools
3.B	Apply numerical results in a given mathematical or applied context.
3.C	Support conclusions or choices with a logical rationale or appropriate data.

Instructional Focus

Unit Enduring Understandings

- Analyzing trends predict behavior.
- Relationships vary by situation.
- Changing one or many pieces can change the whole.
- Procedures can be reversed.
- Logarithmic functions and exponential functions are inverses of one another.
- Appreciation and depreciation affect the rate of growth.

Unit Essential Questions

- How can I make a single model that merges the interest I earn from my bank with the taxes that are due, so I can know how much I will have in the end?

- How can we adjust the scale of distance for a model of planets in the solar system, so the relationships among the planets are easier to see?
- If different functions can be used to model data, how do we pick which one is best?

Objectives

Students will know:

- If f and g are functions, the composite function f◦g maps a set of input values to a set of output values such that the output values of g are used as the input values of f. This is also written as f(g(x)).
- Values of the composite function can be calculated or estimated from the graphical, numerical, analytical, or verbal representations of f and g.
- The composition of functions is not commutative; f◦g does not necessarily equal g◦f.
- Function composition is useful for relating two quantities that are not directly related by an existing formula.
- Functions given analytically can often be decomposed into less complicated functions. When properly decomposed, the variable in one function should replace each instance of the function with which it was composed.
- An additive transformation of a function, f, that results in vertical and horizontal translations can be understood as the composition of g(x) = x + k with f.
- A multiplicative transformation of a function, f, that results in vertical and horizontal dilations can be understood as the composition of g(x) = kx with f. .
- On a specified domain, a function, f, has an inverse function, or is invertible, if each output value of f is mapped from a unique input value. The domain of a function may be restricted in many ways to make the function invertible.
- An inverse function can be thought of as a reverse mapping of the function. An inverse function, f-1(x), maps the output values of a function, f, on its invertible domain to their corresponding input values; that is, if f(a) = b, then f-1(b) = a . Alternatively, on its invertible domain, if a function consists of input-output pairs (a, b), then the inverse function consists of input-output pairs (b, a).
- The composition of a function, f, and its inverse function, f-1, is the identity function; that is, f(f-1(x)) = x and f-1(f(x)) = x.
- On a function's invertible domain, the function's range and domain are the inverse function's domain and range, respectively. The inverse of the table of values of y = f(x) can be found by reversing the input-output pairs. That is, (a, b) corresponds to (b, a).
- In addition to limiting the domain of a function to obtain an inverse function, contextual restrictions may also limit the applicability of an inverse function.
- The logarithmic expression logb(c) is equal to, or represents, the value that the base b must be exponentially raised to in order to obtain the value of c. That is, logb(c) = a if and only if ba = c, where a and c are constants and b > 0 and b =/= 1.
- The values of some logarithmic expressions are readily accessible through basic arithmetic while other values can be estimated through the use of technology.
- On a logarithmic scale, each unit represents a multiplicative change of the base of the logarithm.
- The general form of a logarithmic function is f(x) = alogb(x), with base b, where b > 0, b =/= 1, and a =/= 0.
- The way in which input and output values vary together have an inverse relationship in exponential and logarithmic functions. Output values of general-form exponential functions change proportionately as input values increase in equal-length intervals. However, input values of general-form logarithmic functions change proportionately as output values increase in equal-length intervals. Alternately, exponential growth is characterized by output values changing multiplicatively as input values change additively, whereas logarithmic growth is characterized by output values changing additively as input values change multiplicatively.
- f(x) = logb(x) and g(x) = bx are inverse functions. That is, f(g(x)) = g(f(x)) = x, and the graph of f(x) is a reflection of the graph of g(x) over the line y = x, and the graph of g(x) is a reflection of the graph of f(x) over the line y = x.
- The domain of a logarithmic function in general form is any real number greater than zero, and its range is all real numbers.

- Because logarithmic functions are inverses of exponential functions, logarithmic functions are also always increasing or always decreasing, and their graphs are either always concave up or always concave down. Consequently, logarithmic functions do not have extrema except on a closed interval, and their graphs do not have points of inflection.
- The additive transformation function g(x) = f(x+k) of a logarithmic function f in general form does not have input values that are proportional over equal-length output-value intervals. However, if the input values of the additive transformation function, g(x) = f(x) + k of any function f are proportional over equal-length output value intervals, then f is logarithmic.
- With their limited domain, logarithmic functions in general form are vertically asymptotic to x = 0, with an end behavior that is unbounded.
- The product property for logarithms states that logb(xy) = logb(x) + logb(y).
- The power property for logarithms states that logb(xn) = nlogb(x).
- The change-of-base property for logarithms states that logb(x) = loga(x)/loga(b), where a > 0 and a =/= 1.
- The function ln(x), the natural logarithm, is a logarithmic function with base e. That is, ln(x) = loge(x).
- The function log(x), the common logarithm, is a logarithmic function with base 10. That is, log(x) = log10(x).
- Properties of exponents, properties of logarithms, and the inverse relationship between exponential and logarithmic functions can be used to solve equations and inequalities involving exponents and logarithms.
- When solving exponential and logarithmic equations found through analytical or graphical methods, the results should be examined for extraneous solutions precluded by the mathematical or contextual limitations.
- Logarithms can be used to rewrite expressions involving exponential functions in different ways that may reveal helpful information.
- Logarithmic functions are inverses of exponential functions and can be used to model situations involving proportional growth, or repeated multiplication, where the input values change proportionally over equal-length output-value intervals. Alternatively, if the output value is a whole number, it indicates how many times the initial value has been multiplied by the proportion.
- A logarithmic function model can be constructed from an appropriate proportion and a real zero or from two input-output pairs.
- Logarithmic function models can be constructed for a data set with technology using logarithmic regressions.
- The natural logarithm function is often useful in modeling real-world phenomena.
- Logarithmic function models can be used to predict values for the dependent variable.
- In a semi-log plot, one of the axes is logarithmically scaled. When the y-axis of a semi-log plot is logarithmically scaled, data or functions that demonstrate exponential characteristics will appear linear.
- An advantage of semi-log plots is that a constant never needs to be added to the dependent variable values to reveal that an exponential model is appropriate.
- Techniques used to model linear functions can be applied to a semi-log graph.

Vocabulary: Exponential Growth, Exponential Decay, Compound interest, number e, term, compounding method, Annually, Quarterly, asymptote, logistic, intercept, initial, final, time, Principal, half life, radioactive, Newton's law, Logarithmic, inverse, ph, intensity, laws of log, natural log

Students will be able to:

- Evaluate the composition of two or more functions for given values.
- Construct a representation of the composition of two or more functions.
- Rewrite a given function as a composition of two or more functions.
- Determine the input-output pairs of the inverse of a function.
- Determine the inverse of a function on an invertible domain.
- Evaluate logarithmic expressions.
- Construct representations of the inverse of an exponential function with an initial value of 1.
- Identify key characteristics of logarithmic functions.
- Rewrite logarithmic expressions in equivalent forms.
- Solve exponential and logarithmic equations and inequalities.

- Construct the inverse function for exponential and logarithmic functions.
- Construct a logarithmic function model.
- Determine if an exponential model is appropriate by examining a semi-log plot of a data set.
- Construct the linearization of exponential data.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, Numerical, Algebraic*. Pearson.
Suggested Resources: AP Classroom (2.7-2.15), Desmos, Geogebra, Albert.io

UNIT 3a: Sine, Cosine, and Tangent: Functions, Graphs, and Transformations

Summary and Rationale

In Unit 3, students explore trigonometric functions and their relation to the angles and arcs of a circle. Since their output values repeat with every full revolution around the circle, trigonometric functions are ideal for modeling periodic, or repeated pattern phenomena, such as: the highs and lows of a wave, the blood pressure produced by a heart, and the angle from the North Pole to the Sun year to year. Furthermore, periodicity is found in human inventions and social phenomena. For example, moving parts of an analog clock are modeled by a trigonometric function with respect to each other or with respect to time; traffic flow at an intersection over the course of a week demonstrates daily periodicity; and demand for a particular product over the course of a year falls into an annually repeating pattern. Polar functions, which are also explored in this unit, have deep ties to trigonometric functions as they are both based on the circle. Polar functions are defined on the polar coordinate system that uses the circular concepts of radii and angles to describe location instead of rectangular concepts of left-right and up-down, which students have worked with previously. Trigonometry serves as the bridge between the two systems.

Recommended Pacing

9 Blocks

AP Mathematical Practices

Practice 1 - Procedural and Symbolic Fluency	
1.A	Solve equations and inequalities represented analytically, with and without technology.
1.B	Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C	Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.
Practice 2 - Multiple Representations	
2.A	Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B	Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
Practice 3 - Communication and Reasoning	
3.A	Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools
3.B	Apply numerical results in a given mathematical or applied context.
3.C	Support conclusions or choices with a logical rationale or appropriate data.

Instructional Focus

Unit Enduring Understandings

- Trigonometric functions are used to model periodic behavior.
- Look for and make use of structure in the graphical and algebraic representation of trigonometric functions.
- Interpret key features of a trigonometric function algebraically and graphically.
- Use the concept of a function to identify the domain restrictions for inverse trigonometric functions.

Unit Essential Questions

- Since energy usage goes up and down throughout the year, how can I use trends in data to predict my monthly electricity bills when I get my first apartment?
- How do we model aspects of circular and spinning objects without using complex equations from the xy rectangular-based coordinate system?
- How does right triangle trigonometry from geometry relate to trigonometric functions?

Objectives

Students will know:

- A periodic relationship can be identified between two aspects of a context if, as the input values increase, the output values demonstrate a repeating pattern over successive equal-length intervals.
- The graph of a periodic relationship can be constructed from the graph of a single cycle of the relationship.
- The period can be estimated by investigating successive equal-length output values and finding where the pattern begins to repeat.
- Periodic functions take on characteristics of other functions, such as intervals of increase and decrease, different concavities, and various rates of change. However, with periodic functions, all characteristics found in one period of the function will be in every period of the function.
- In the coordinate plane, an angle is in standard position when the vertex coincides with the origin and one ray coincides with the positive x-axis. The other ray is called the terminal ray. Positive and negative angle measures indicate rotations from the positive x-axis in the counterclockwise and clockwise direction, respectively. Angles in standard position that share a terminal ray differ by an integer number of revolutions.
- The radian measure of an angle in standard position is the ratio of the length of the arc of a circle centered at the origin subtended by the angle to the radius of that same circle. For a unit circle, which has radius 1, the radian measure is the same as the length of the subtended arc.
- Given an angle in standard position and a circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The sine of the angle is the ratio of the vertical displacement of P from the x-axis to the distance between the origin and point P. Therefore, for a unit circle, the sine of the angle is the y-coordinate of point P.
- Given an angle in standard position and a circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The cosine of the angle is the ratio of the horizontal displacement of P from the y-axis to the distance between the origin and point P. Therefore, for a unit circle, the cosine of the angle is the x-coordinate of point P.
- Given an angle in standard position, the tangent of the angle is the slope, if it exists, of the terminal ray. Because the slope of the terminal ray is the ratio of the vertical displacement to the horizontal displacement over any interval, the tangent of the angle is the ratio of the y-coordinate to the x-coordinate of the point at which the terminal ray intersects the unit circle; alternately, it is the ratio of the angle's sine to its cosine.
- Given an angle measure θ in standard position and a circle with radius r centered at the origin, there is a point P where the terminal ray intersects the circle. The coordinates of point P are $(r\cos\theta, r\sin\theta)$.
- The geometry of isosceles right and equilateral triangles, while attending to the signs of the values based on the quadrant of the angle, can be used to find exact values for the cosine and sine of angles that are multiples of $\pi/4$ and $\pi/6$ radians and whose terminal rays do not lie on an axis.
- As the input values, or angle measures, of the sine function increase, the output values oscillate between -1 and 1, taking every value in between and tracking the vertical distance of points on the unit circle from the x-axis.
- As the input values, or angle measures, of the cosine function increase, the output values oscillate between -1 and 1, taking every value in between and tracking the horizontal distance of points on the unit circle from the y-axis.
- A sinusoidal function is any function that involves additive and multiplicative transformations of $f(\theta) = \sin(\theta)$. This includes $\cos(\theta) = \sin(\theta + \pi/2)$.
- The period and frequency of a sinusoidal function are reciprocals. The period of $f(\theta) = \sin(\theta)$ and $g(\theta) = \cos(\theta)$ is 2π, and the frequency is $1/(2\pi)$.

- The amplitude of a sinusoidal function is half the difference between its maximum and minimum values. The amplitude of f(θ) = sin(θ) and g(θ) = cos(θ) is 1.
- The midline of the graph of a sinusoidal function is determined by the average, or arithmetic mean, of the maximum and minimum values of the function. The midline of the graphs of f(θ) = sin(θ) and g(θ) = cos(θ) is y = 0, the x-axis.
- As input values increase, the graphs of sinusoidal functions oscillate between concave down and concave up.
- The graph of y = sin(x) has rotational symmetry about the origin and is therefore an odd function. The graph of y = cos(θ) has reflective symmetry over the y-axis and is therefore an even function.
- Functions that can be written in the form f(θ) = asin(b(θ+c))+d or g(θ) = acos(b(θ+c))+d, where a, b, c, and d are real numbers and a =/= 0, are sinusoidal functions and are transformations of the sine and cosine functions.
- The smallest interval of input values over which the maximum or minimum output values start to repeat, that is, the input-value interval between consecutive maxima or consecutive minima, can be used to determine or estimate the period and frequency for a sinusoidal function model.
- The maximum and minimum output values can be used to determine or estimate the amplitude and vertical shift for a sinusoidal function model.
- An actual pair of input-output values can be compared to pairs of input-output values produced by a sinusoidal function model to determine or estimate a phase shift for the model.
- Sinusoidal function models can be constructed for a data set with technology by estimating key values or using sinusoidal regressions.
- Sinusoidal functions that model a data set are frequently only useful over their contextual domain and can be used to predict values of the dependent variable from values of the independent variable.
- Because the slope of the terminal ray is the ratio of the change in the y-values to the change in the x-values between any two points on the ray, the tangent function is also the ratio of the sine function to the cosine function. Therefore, tan(θ) = sin(θ)/cos(θ), where cos(θ) =/= 0.
- Because the slope values of the terminal ray repeat every one-half revolution of the circle, the tangent function has a period of π.
- The tangent function increases and its graph changes from concave down to concave up between consecutive asymptotes.
- Functions that can be written in the form f(θ) = atan(b(θ+c))+d are transformations of the tangent function.

Vocabulary: Amplitude, phase shift, trigonometry, period, sine, cosine, tangent, cotangent, cosecant, secant, vertical shift, unit circle, reference number, terminal, sinusoidal

Students will be able to:
- Construct graphs of periodic relationships based on verbal representations.
- Describe key characteristics of a periodic function based on a verbal representation.
- Determine the sine, cosine, and tangent of an angle using the unit circle.
- Determine coordinates of points on a circle centered at the origin.
- Construct representations of the sine and cosine functions using the unit circle.
- Identify key characteristics of the sine and cosine functions.
- Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.
- Construct sinusoidal function models of periodic phenomena.
- Construct representations of the tangent function using the unit circle.
- Describe key characteristics of the tangent function.
- Describe additive and multiplicative transformations involving the tangent function.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, Numerical, Algebraic*. Pearson.
Suggested Resources: AP Classroom (3.1-3.8), Desmos, Geogebra, Albert.io

UNIT 3b: Inverse Trigonometry; Trigonometric Equations; and Cosecant, Secant, and Cotangent

Summary and Rationale	
In Unit 3, students explore trigonometric functions and their relation to the angles and arcs of a circle. Since their output values repeat with every full revolution around the circle, trigonometric functions are ideal for modeling periodic, or repeated pattern phenomena, such as: the highs and lows of a wave, the blood pressure produced by a heart, and the angle from the North Pole to the Sun year to year. Furthermore, periodicity is found in human inventions and social phenomena. For example, moving parts of an analog clock are modeled by a trigonometric function with respect to each other or with respect to time; traffic flow at an intersection over the course of a week demonstrates daily periodicity; and demand for a particular product over the course of a year falls into an annually repeating pattern. Polar functions, which are also explored in this unit, have deep ties to trigonometric functions as they are both based on the circle. Polar functions are defined on the polar coordinate system that uses the circular concepts of radii and angles to describe location instead of rectangular concepts of left-right and up-down, which students have worked with previously. Trigonometry serves as the bridge between the two systems.	
Recommended Pacing	
6 Blocks	
AP Mathematical Practices	
Practice 1 - Procedural and Symbolic Fluency	
1.A	Solve equations and inequalities represented analytically, with and without technology.
1.B	Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C	Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.
Practice 2 - Multiple Representations	
2.A	Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B	Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
Practice 3 - Communication and Reasoning	
3.A	Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools
3.B	Apply numerical results in a given mathematical or applied context.
3.C	Support conclusions or choices with a logical rationale or appropriate data.
Instructional Focus	
Unit Enduring Understandings	
• The skills used to manipulate algebraic expressions are needed to simplify trigonometric expressions • The trigonometric identities can be written in multiple equivalent forms	
Unit Essential Questions	

- Since energy usage goes up and down throughout the year, how can I use trends in data to predict my monthly electricity bills when I get my first apartment?
- How do we model aspects of circular and spinning objects without using complex equations from the xy rectangular-based coordinate system?
- How does right triangle trigonometry from geometry relate to trigonometric functions?

Objectives

Students will know:

- For inverse trigonometric functions, the input and output values are switched from their corresponding trigonometric functions, so the output value of an inverse trigonometric function is often interpreted as an angle measure and the input is a value in the range of the corresponding trigonometric function.
- The inverse trigonometric functions are also called arcsine, arccosine, and arctangent, and can be represented as sin-1(x), cos-1(x), and tan-1(x), respectively. Because the corresponding trigonometric functions are periodic, they are only invertible if they have restricted domains.
- In order to define their respective inverse functions, the domain of the sine function is restricted to [-π/2, π/2], the cosine function to [0, π], and the tangent function to (-π/2, π/2).
- Inverse trigonometric functions are useful in solving equations and inequalities involving trigonometric functions, but solutions may need to be modified due to domain restrictions.
- Because trigonometric functions are periodic, there are often infinitely many solutions to trigonometric equations.
- In trigonometric equations and inequalities arising from a contextual scenario, there is often a domain restriction that can be implied from the context, which limits the number of solutions.
- The cosecant function, f(θ) = csc(θ), is the reciprocal of the sine function, where sin(θ) =/= 0.
- The secant function, f(θ) = sec(θ), is the reciprocal of the cosine function, where cos(θ) =/= 0.
- The cotangent function, f(θ) = cot(θ), is the reciprocal of the tangent function, where tan(θ) =/= 0.
- The graph of the cotangent function has vertical asymptotes for domain values where tan(θ) = 0 and is decreasing between consecutive asymptotes.
- The Pythagorean Theorem can be applied to right triangles with points on the unit circle at coordinates (cos(θ), sin(θ)), resulting in the Pythagorean identity sin2(θ) + cos2(θ) = 1.
- The sum identity for sine is sin(a+b) = sin(a)cos(b) + cos(a)sin(b).
- The sum identity for cosine is cos(a+b) = cos(a)cos(b) - sin(a)sin(b).
- The sum identities for sine and cosine can also be used as difference and double-angle identities.
- Properties of trigonometric functions, known trigonometric identities, and other algebraic properties can be used to verify additional trigonometric identities.
- A specific equivalent form involving trigonometric expressions can make information more accessible.
- Equivalent trigonometric forms may be useful in solving trigonometric equations and inequalities.

Vocabulary: Identity, verify, Pythagorean, odd/even, cofunction, sum and difference, Amplitude, phase shift, trigonometry, period, sine, cosine, tangent, cotangent, cosecant, secant, vertical shift, unit circle, reference number, terminal, sinusoidal

Students will be able to:

- Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain.
- Solve equations and inequalities involving trigonometric functions.
- Identify key characteristics of functions that involve quotients of the sine and cosine functions.
- Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity.
- Rewrite trigonometric expressions in equivalent forms with sine and cosine sum identities.
- Solve equations using equivalent analytic representations of trigonometric functions.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, Numerical, Algebraic*. Pearson.
Suggested Resources: AP Classroom (3.9-3.12), Desmos, Geogebra, Albert.io

UNIT 3c: Polar Coordinates and Polar Functions

Summary and Rationale

In Unit 3, students explore trigonometric functions and their relation to the angles and arcs of a circle. Since their output values repeat with every full revolution around the circle, trigonometric functions are ideal for modeling periodic, or repeated pattern phenomena, such as: the highs and lows of a wave, the blood pressure produced by a heart, and the angle from the North Pole to the Sun year to year. Furthermore, periodicity is found in human inventions and social phenomena. For example, moving parts of an analog clock are modeled by a trigonometric function with respect to each other or with respect to time; traffic flow at an intersection over the course of a week demonstrates daily periodicity; and demand for a particular product over the course of a year falls into an annually repeating pattern. Polar functions, which are also explored in this unit, have deep ties to trigonometric functions as they are both based on the circle. Polar functions are defined on the polar coordinate system that uses the circular concepts of radii and angles to describe location instead of rectangular concepts of left-right and up-down, which students have worked with previously. Trigonometry serves as the bridge between the two systems.

Recommended Pacing

5 Blocks

AP Mathematical Practices

Practice 1 - Procedural and Symbolic Fluency	
1.A	Solve equations and inequalities represented analytically, with and without technology.
1.B	Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C	Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.
Practice 2 - Multiple Representations	
2.A	Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B	Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
Practice 3 - Communication and Reasoning	
3.A	Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools
3.B	Apply numerical results in a given mathematical or applied context.
3.C	Support conclusions or choices with a logical rationale or appropriate data.
Practice 1 - Procedural and Symbolic Fluency	

Instructional Focus

Unit Enduring Understandings

- Analytic geometry is a systematic approach that can be used to solve algebra problems geometrically and geometry problems algebraically

Unit Essential Questions

- Since energy usage goes up and down throughout the year, how can I use trends in data to predict my monthly electricity bills when I get my first apartment?
- How do we model aspects of circular and spinning objects without using complex equations from the xy rectangular-based coordinate system?
- How does right triangle trigonometry from geometry relate to trigonometric functions?

Objectives

Students will know:

- The polar coordinate system is based on a grid of circles centered at the origin and on lines through the origin. Polar coordinates are defined as an ordered pair (r, θ), such that |r| represents the radius of the circle on which the point lies, and θ represents the measure of an angle in standard position whose terminal ray includes the point. In the polar coordinate system, the same point can be represented in many ways.
- The coordinates of a point in the polar coordinate system, (r, θ), can be converted to coordinates in the rectangular coordinate system, (x, y), such that x = rcos(θ) and y = rsin(θ).
- The coordinates of a point in the rectangular coordinate system, (x, y), can be converted to coordinates in the polar coordinate system, (r, θ), using r = √(x2+y2) and θ = arctan(y/x) for x > 0 and θ = arctan(y/x) + π for x < 0.
- A complex number can be understood as a point in the complex plane and can be determined by its corresponding rectangular or polar coordinates. When the complex number has the rectangular coordinates (a, b), it can be expressed as a + bi. When the complex number has polar coordinates (r, θ), it can be expressed as (rcos(θ)) + i(rsin(θ)).
- The graph of the function r = f(θ) in polar coordinates consists of input-output pairs of values where the input values are angle measures and the output values are radii.
- The domain of the polar function r = f(θ), given graphically, can be restricted to a desired portion of the function by selecting endpoints corresponding to the desired angle and radius.
- When graphing polar functions in the form r = f(θ), changes in input values correspond to changes in angle measure from the positive x-axis, and changes in output values correspond to changes in distance from the origin.
- If a polar function r = f(θ) is positive and increasing or negative and decreasing, then the distance between f(θ) and the origin is increasing.
  If a polar function r = f(θ) is positive and decreasing or negative and increasing, then the distance between f(θ) and the origin is decreasing.
  For a polar function r = f(θ), if the function changes from increasing to decreasing or decreasing to increasing on an interval, then the function has a relative extremum on the interval corresponding to a point relatively closest to or farthest from the origin.
- The average rate of change of r with respect to θ over an interval of θ is the ratio of the change in the radius values to the change in θ over an interval of θ. Graphically, the average rate of change indicates the rate at which the radius is changing per radian.
- The average rate of change of r with respect to θ over an interval of θ can be used to estimate values of the function within the interval.

Vocabulary: polar, rectangular coordinate, arc, cardioid, limacon, polar grid.

Students will be able to:

- Determine the location of a point in the plane using both rectangular and polar coordinates.
- Construct graphs of polar functions.
- Describe characteristics of the graph of a polar function.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, Numerical, Algebraic*. Pearson. **Suggested Resources:** AP Classroom (3.13-3.15), Desmos, Geogebra, Albert.io

UNIT 4a: Parametrics and Conics

Summary and Rationale

In Unit 4, students explore function types that expand their understanding of the function concept. Parametric functions have multiple dependent variables' values paired with a single input variable or parameter. Modeling scenarios with parametric functions allows students to explore change in terms of components. This component-based understanding is important not only in calculus but in all fields of the natural and social sciences where we seek to understand one aspect of a phenomenon independent of other confounding aspects. Another major function type in this unit involves matrices mapping a set of input vectors to output vectors. The capacity to map large quantities of vectors instantaneously is the basis for vector-based computer graphics. While students may see their favorite video game character trip and fall or seemingly move closer or farther, matrices implement a rotation on a set of vectors or a dilation on a set of vectors. The power of matrices to map vectors is not limited to graphics but to any system that can be expressed in terms of components of vectors such as electrical systems, network connections, and regional population distribution changes over time. Vectors and matrices are also powerful tools of data science as they can be used to model aspects of complex scientific and social science phenomena.

Recommended Pacing

5 Blocks

AP Mathematical Practices

Practice 1 - Procedural and Symbolic Fluency

1.A	Solve equations and inequalities represented analytically, with and without technology.
1.B	Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C	Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.

Practice 2 - Multiple Representations

2.A	Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B	Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.

Practice 3 - Communication and Reasoning

3.A	Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools
3.B	Apply numerical results in a given mathematical or applied context.
3.C	Support conclusions or choices with a logical rationale or appropriate data.

Instructional Focus

Unit Enduring Understandings

- Parametric equations are separate equations for each of the dimension
- The variable in the equation is called the parameter and is denoted by t or theta.

Unit Essential Questions

- How can we determine when the populations of species in an ecosystem will be relatively steady?
- How can we analyze the vertical and horizontal aspects of motion independently?
- How does high-resolution computer-generated imaging achieve smooth and realistic motion on screen with so many pixels?

Objectives

Students will know:

- A parametric function in R2, the set of all ordered pairs of two real numbers, consists of a set of two parametric equations in which two dependent variables, x and y, are dependent on a single independent variable, t, called the parameter.
- Because the variables x and y are dependent on t, the coordinates (xi, yi) at time ti can be written as functions of t and can be expressed as the single parametric function f(t) = (x(t), y(t)), where x and y are the names of two functions.
- A numerical table of values can be generated for the parametric function f(t) = (x(t), y(t)).
- A graph of a parametric function can be sketched by connecting several points from the numerical table of values in order of increasing value of t.
- The domain of the parametric function f is often restricted, which results in start and end points on the graph of f.
- A parametric function given by f(t) = (x(t), y(t)) can be used to model particle motion in the plane. The graph of this function indicates the position of a particle at time t.
- The horizontal and vertical extrema of a particle's motion can be determined by identifying the maximum and minimum values of the functions x(t) and y(t), respectively.
- The real zeros of the function x(t) correspond to y-intercepts, and the real zeros of y(t) correspond to x-intercepts.
- As the parameter increases, the direction of planar motion of a particle can be analyzed in terms of x and y independently. If x(t) is increasing or decreasing, the direction of motion is to the right or left, respectively. If y(t) is increasing or decreasing, the direction of motion is up or down, respectively.
- At any given point in the plane, the direction of planar motion may be different for different values of t.
- The same curve in the plane can be parametrized in different ways and can be traversed in different directions with different parametric functions.
- A complete counterclockwise revolution around the unit circle that starts and ends at (1,0) and is centered around the origin can be modeled by (x(t), y(t)) = (cos(t), sin(t)) with domain 0 ≤ t ≤ 2π.
- Transformations of the parametric function (x(t), y(t)) = (cos(t), sin(t)) can model any circular path traversed in the plane.
- A linear path along the line segment from the point (x1, y1) to the point (x2, y2) can be parametrized many ways, including using an initial position (x1, y1) and rates of change for x with respect to t and y with respect to t.
- An equation involving two variables can implicitly describe one or more functions.
- An equation involving two variables can be graphed by finding solutions to the equation.
- Solving for one of the variables in an equation involving two variables can define a function whose graph is part or all of the graph of the equation.
- For ordered pairs on the graph of an implicitly defined function that are close together, if the ratio of the change in the two variables is positive, then the two variables simultaneously increase or both decrease; conversely, if the ratio is negative, then as one variable increases, the other decreases.
- The rate of change of x with respect to y or of y with respect to x can be zero, indicating vertical or horizontal intervals, respectively.
- A parabola with vertex (h,k) can, if a =/= 0, be represented analytically as x – h = a(y-k)2 if it opens left or right, or as y – k = a(x-h)2 if it opens up or down.
- An ellipse centered at (h,k) with horizontal radius a and vertical radius b can be represented analytically as (x-h)2/a2 + (y-k)2/b2 = 1. A circle is a special case of an ellipse where a = b.

- A hyperbola centered at (h,k) with vertical and horizontal lines of symmetry can be represented algebraically $(x-h)2/a2 - (y-k)2/b2 = 1$ for a hyperbola opening left and right, or as $(y-k)2/b2 - (x-h)2/a2 = 1$ for a hyperbola opening up and down. The asymptotes are $y-k = \pm(b/a)(x-h)$.
- A parametrization $(x(t), y(t))$ or an implicitly defined function will, when $x(t)$ and $y(t)$ are substituted for x and y, respectively, satisfy the corresponding equation for every value of t in the domain.
- A parabola can be parametrized in the same way that any equation that can be solved for x or y can be parametrized. Equations that can be solved for x can be parametrized as $(x(t), y(t)) = (f(t), t)$ by solving for x and replacing y with t. Equations that can be solved for y can be parametrized as $(x(t), y(t)) = (t, f(t))$ by solving for y and replacing x with t.
- An ellipse can be parametrized using the trigonometric functions $x(t) = h + a\cos(t)$ and $y(t) = k + b\sin(t)$ for $0 \le t \le 2\pi$.
- A hyperbola can be parametrized using trigonometric functions. For a hyperbola that opens left and right, the functions are $x(t) = h + a\sec(t)$ and $y(t) = k + b\tan(t)$ for $0 \le t \le 2\pi$. For a hyperbola that opens up and down, the functions are $x(t) = h + a\tan(t)$ and $y(t) = k + b\sec(t)$ for $0 \le t \le 2\pi$.

Vocabulary: Conics, parametric, circle, curve,plane, motion, planar, restricted, hyperbola, ellipse, parabola, implicitly

Students will be able to:
- Construct a graph or table of values for a parametric function represented analytically.
- Identify key characteristics of a parametric planar motion function that are related to position.
- Identify key characteristics of a parametric planar motion function that are related to direction and rate of change.
- Express motion around a circle or along a line segment parametrically.
- Construct a graph of an equation involving two variables.
- Determine how the two quantities related in an implicitly defined function vary together.
- Represent conic sections with horizontal or vertical symmetry analytically.
- Represent a curve in the plane parametrically.
- Represent conic sections parametrically.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, Numerical, Algebraic*. Pearson.
Suggested Resources: AP Classroom (4.1-4.7), Desmos, Geogebra, Albert.io

UNIT 4b: Vectors and Matrices

Summary and Rationale	
In Unit 4, students explore function types that expand their understanding of the function concept. Parametric functions have multiple dependent variables' values paired with a single input variable or parameter. Modeling scenarios with parametric functions allows students to explore change in terms of components. This component-based understanding is important not only in calculus but in all fields of the natural and social sciences where we seek to understand one aspect of a phenomenon independent of other confounding aspects. Another major function type in this unit involves matrices mapping a set of input vectors to output vectors. The capacity to map large quantities of vectors instantaneously is the basis for vector-based computer graphics. While students may see their favorite video game character trip and fall or seemingly move closer or farther, matrices implement a rotation on a set of vectors or a dilation on a set of vectors. The power of matrices to map vectors is not limited to graphics but to any system that can be expressed in terms of components of vectors such as electrical systems, network connections, and regional population distribution changes over time. Vectors and matrices are also powerful tools of data science as they can be used to model aspects of complex scientific and social science phenomena.	
Recommended Pacing	
5 Blocks	
AP Mathematical Practices	
Practice 1 - Procedural and Symbolic Fluency	
1.A	Solve equations and inequalities represented analytically, with and without technology.
1.B	Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C	Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.
Practice 2 - Multiple Representations	
2.A	Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B	Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
Practice 3 - Communication and Reasoning	
3.A	Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools
3.B	Apply numerical results in a given mathematical or applied context.
3.C	Support conclusions or choices with a logical rationale or appropriate data.
Instructional Focus	
Unit Enduring Understandings	
• Systems of equations can be represented and solved using matrices • Represent speed, force, bearing using Vectors • Solve speed, force, bearing problems using resultant vectors.	
Unit Essential Questions	

- How can we determine when the populations of species in an ecosystem will be relatively steady?
- How can we analyze the vertical and horizontal aspects of motion independently?
- How does high-resolution computer-generated imaging achieve smooth and realistic motion on screen?

Objectives

Students will know:

- A vector is a directed line segment. When a vector is placed in the plane, the point at the beginning of the line segment is called the tail, and the point at the end of the line segment is called the head. The length of the line segment is the magnitude of the vector.
- A vector P1P2 with two components can be plotted in the xy-plane from P1 = (x1, y1) to P2 = (x2, y2). The vector is identified by a and b, where a = x2 – x1 and b = y2 – y1. The vector can be expressed as <a, b>. A zero vector <0, 0> is the trivial case when P1 = P2.
- The direction of the vector is parallel to the line segment from the origin to the point with coordinates (a, b). The magnitude of the vector is the square root of the sum of the squares of the components.
- For a vector represented geometrically in the plane, the components of the vector can be found using trigonometry
- The multiplication of a constant and a vector results in a new vector whose components are found by multiplying the constant by each of the components of the original vector. The new vector is parallel to the original vector.
- The sum of two vectors in R2 is a new vector whose components are found by adding the corresponding components of the original vectors. The new vector can be represented graphically as a vector whose tail corresponds to the tail of the first vector and whose head corresponds to the head of the second vector when the second vector's tail is located at the first vector's head.
- The dot product of two vectors is the sum of the products of their corresponding components. That is, <a1, b1> * <a2, b2> = a1a2 + b1b2.
- A unit vector is a vector of magnitude 1. A unit vector in the same direction as a given nonzero vector can be found by scalar multiplying the vector by the reciprocal of its magnitude.
- The vector <a, b> can be expressed as ai + bj in R2, where i and j are unit vectors in the x and y directions, respectively. That is, i = <1, 0> and j = <0, 1>.
- The dot product is geometrically equivalent to the product of the magnitudes of the two vectors and the cosine of the angle between them. Therefore, if the dot product of two nonzero vectors is zero, then the vectors are perpendicular.
- The Law of Sines and Law of Cosines can be used to determine side lengths and angle measures of triangles formed by vector addition.
- The position of a particle moving in a plane that is given by the parametric function f(t) = (x(t), y(t)) may be expressed as a vector-valued function, p(t) = ix(t) + jy(t) or p(t) = <x(t), y(t)>. The magnitude of the position vector at time t gives the distance of the particle from the origin.
- The vector-valued function v(t) = <x(t), y(t)> can be used to express the velocity of a particle moving in a plane at different times, t. At time t, the sign of x(t) indicates if the particle is moving right or left, and the sign of y(t) indicates if the particle is moving up or down. The magnitude of the velocity vector at time t gives the speed of the particle.
- An n x m matrix is an array consisting of n rows and m columns.
- Two matrices can be multiplied if the number of columns in the first matrix equals the number of rows in the second matrix. The product of the matrices is a new matrix in which the component in the ith row and jth column is the dot product of the ith row of the first matrix and the jth column of the second matrix.
- The identity matrix, I, is a square matrix consisting of 1s on the diagonal from the top left to bottom right and 0s everywhere else.
- Multiplying a square matrix by its corresponding identity matrix results in the original square matrix.
- The product of a square matrix and its inverse, when it exists, is the identity matrix of the same size.
- The inverse of a 2 x 2 matrix, when it exists, can be calculated with or without technology.

- The determinant of the 2 x 2 matrix A, with top row a, b and bottom row c, d, is ad – bc. The determinant can be calculated with or without technology and is denoted det(A).
- If a 2 x 2 matrix consists of two column or row vectors from R2 then the nonzero absolute value of the determinant of the matrix is the area of the parallelogram spanned by the vectors represented in the columns or rows of the matrix. If the determinant equals 0, then the vectors are parallel.
- The square matrix A has an inverse if and only if det(A) =/= 0.
- A linear transformation is a function that maps an input vector to an output vector such that each component of the output vector is the sum of constant multiples of the input vector components.
- A linear transformation will map the zero vector to the zero vector.
- The mapping of the unit vectors in a linear transformation provides valuable information for determining the associated matrix.
- A 2 x 2 matrix with top row cos(θ), -sin(θ) and bottom row sin(θ), cos(θ) is associated with a linear transformation of vectors that rotates every vector an angle θ counterclockwise about the origin.
- The absolute value of the determinant of a 2 x 2 transformation matrix gives the magnitude of the dilation of regions in R2 under the transformation.
- The composition of two linear transformations is a linear transformation.
- The matrix associated with the composition of two linear transformations is the product of the matrices associated with each linear transformation.
- Two linear transformations are inverses if their composition maps any vector to itself.
- A contextual scenario can indicate the rate of transitions between states as percent changes. A matrix can be constructed based on these rates to model how states change over discrete intervals.
- The product of a matrix that models transitions between states and a corresponding state vector can predict future states.
- Repeated multiplication of a matrix that models the transitions between states and corresponding resultant state vectors can predict the steady state, a distribution between states that does not change from one step to the next.
- The product of the inverse of a matrix that models transitions between states and a corresponding state vector can predict past states.

Vocabulary: Matrix, determinant, product, inverse of matrix, row echelon form, vectors, resultant, magnitude, vector addition, unit vector

Students will be able to:
- Identify characteristics of a vector.
- Determine sums and products involving vectors.
- Determine a unit vector for a given vector.
- Determine angle measures between vectors and magnitudes of vectors involved in vector addition.
- Represent planar motion in terms of vector-valued functions.
- Determine the product of two matrices.
- Determine the inverse of a 2 x 2 matrix.
- Apply the value of the determinant to invertibility and vectors.
- Determine the output vectors of a linear transformation using a 2 x 2 matrix.
- Determine the association between a linear transformation and a matrix.
- Determine the composition of two linear transformations.
- Determine the inverse of a linear transformation.
- Construct a model of a scenario involving transitions between two states using matrices.
- Apply matrix models to predict future and past states for n transition steps.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, Numerical, Algebraic*. Pearson.

Suggested Resources: AP Classroom (4.8-4.14), Desmos, Geogebra, Albert.io