PISCATAWAY TOWNSHIP SCHOOLS

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Discrete Mathematics

Content Area:	Mathematics
Grade Span:	11-12
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COURSE OVERVIEW

Description

Discrete Mathematics is a full-year college preparatory course that will allow students to gain an appreciation of the subtlety and variety of mathematics. The topics are chosen with the purpose of introducing the student to a different view of mathematics from the one presented in a traditional general education mathematics curriculum. The course is focused on building a direct and immediate connection between the mathematics of our world and the concrete, real-life problems in which mathematics is realized.

Goals

In addition to the content standards, skills, and concepts set forth, this course also seeks to meet the Standards for Mathematical Practice. These practices include generally applied best practices for learning mathematics, such as understanding the nature of proof and having a productive disposition towards the subject, and are not tied to a particular set of content. These skills are applicable beyond a student's study of mathematics.

The eight Standards for Mathematical Practice are outlined below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Scope and Sequence

Unit	Topic	Length (90 Blocks)
1	Management Science	35-42
2	Growth and Symmetry	6-8
3	Statistics and Probability	18-20
4	Mathematics of Social Choice	15-20

Resources

Core Text:
Excursions in Modern Mathematics. Peter Tannenbaum. Pearson Education, 2004.

Suggested Resources:
Excel, Google Sheets, Desmos

UNIT 1: Management Science

Summary and Rationale

This Unit deals with methods for solving problems involving the organization and management of complex activities- that is activities involving a large number of steps and/or a large number of variables. Students learn that efficiency is important to all of the problems and constraints must be minimized.

Recommended Pacing

35-42 Blocks

State Standards

Standard N-Q Quantities

CPI #	Cumulative Progress Indicator (CPI)
2	Define appropriate quantities for the purpose of descriptive modeling.

Instructional Focus

Unit Enduring Understandings

- Models help to create connections and make sense of mathematical phenomena.
- Problem-solving teaches efficiency and perseverance.

Unit Essential Questions

- How do the graphs of mathematical models and data help us better understand the world in which we live?
- What do effective problem solvers do, and what do they do when they get stuck?

Objectives

Students will know:

- How to describe relationships within a set of objects using graphs.
- The terminology and concepts associated with graphs and circuits.
- The sum of the degrees of all the vertices of a graph equals twice the number of edges.
- That an algorithm, when followed, provides solutions to graph theory problems.
- The difference between an Euler circuit/path and a Hamilton circuit/path.
- The relationship between a completed graph with N vertices and the number of Hamilton circuits.
- When an optimal Hamilton circuit exists in a weighted graph.
- Various strategies/algorithms for solving traveling-salesman problems and when each is useful to solving traveling-salesman problems.
- The concepts of subgraphs, trees, the properties of trees, spanning trees, and minimum spanning trees.
- Kruskal's algorithm is an optimal algorithm for producing a minimum spanning tree.
- Steiner points and the terminology associated with them.
- The terminology associated with the basic elements of scheduling.
- Data can be represented in an organized graph called a digraph, which incorporates directionality.
- The various algorithms that are associated with finding and optimizing critical paths.

Vocabulary:

adjacent edges, adjacent vertices, algorithms, bridge, circuit, connected graphs, degree, disconnected graphs, edge, Euler circuit, Eulerizign a graph, Euler path, even and odd vertex, Fleury's theorem, graph, loop, path, routing problem, unicursal tracing, vertex, Hamilton circuit, factorial, Hamilton path, weighted graph, minimum spanning tree, network, Steiner point, Steiner tree, subgraph, critical path, arc, cycle, digraph, priority list, processing time, task

Students will be able to:

- Draw graph models from real-life scenarios, recognize when a graph cannot be drawn, and determine if they are traversable.
- Apply Euler's theorems to determine if a graph has an Euler circuit, an Euler path, or neither.
- Apply Fleury's Algorithm for finding an Euler circuit.
- Eulerize a graph by adding additional edges.
- Determine if a graph has a Hamilton circuit or Hamilton path.
- Find an optimal Hamilton circuit, known as a traveling-salesman problem.
- Use various strategies/algorithms to solve traveling-salesman problems, including the Brute-Force Algorithm, the Nearest-Neighbor Algorithm, the Repetitive Nearest-Neighbor Algorithm, and the Cheapest-Link Algorithm.
- Draw trees that represent networks that are connected and have no circuits, then verify the properties of trees apply.
- Apply Kruskal's Algorithm as a variation of the Cheapest-Link Algorithm.
- Use Torricelli's Method for finding a Steiner Point and find the shortest distance between three points.
- Draw and analyze digraphs from given data.
- Apply the Priority-List Model for Scheduling and analyze the results.
- Apply the Decreased-Time Algorithm to create a schedule.
- Apply the Backflow Algorithm by understanding critical paths/times and how working from end to start will optimize the schedule.
- Apply the Critical-Path Algorithm to create a schedule that uses the critical-path list as the priority list.
- Determine the relative percentage of error for a scheduling algorithm that is applied to a particular situation.

Resources

Core Text:

Excursions in Modern Mathematics. Peter Tannenbaum. Pearson Education, 2004. *Chapters 5, 6, 7 and 8*

Suggested Resources:

Excel, Google Sheets, Desmos

UNIT 2: Growth and Symmetry

Summary and Rationale

This unit deals with nontraditional geometric ideas. Students will explore the patterns and behaviors of geometry through the analysis of how sunflowers, seashells, and animal populations grow. They will then look at the symmetries of a snowflake and the patterns in wallpaper. These real-life examples will show students how nontraditional geometry is tied to various growth patterns learned in high-school mathematics.

Recommended Pacing

6-8 blocks

State Standards

Standard F-LE Linear and Exponential Models

CPI #	Cumulative Progress Indicator (CPI)
1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function
4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology
5	Interpret the parameters in a linear or exponential function in terms of a context.

Instructional Focus

Unit Enduring Understandings

- Understanding attributes develop clarity for the system or model.
- Growth models can be determined through the analysis of spatial relationships.

Unit Essential Questions

- What are the mathematical attributes of objects or processes and how are they measured or calculated?
- How are spatial relationships, including shape and dimension, used to draw, construct, model, and represent real situations or solve problems?

Objectives

Students will know:

- The difference between discrete and continuous growth.
- Linear models for population growth occur when each transition is a constant amount.
- Linear growth models represent arithmetic sequences and exponential models represent geometric sequences.
- Exponential models for population growth occur when each transition changes by a fixed proportion.

Vocabulary:
annual yield, arithmetic sequence, common difference, common ratio, continuous growth, explicit description, exponential growth or decay, geometric sequence, linear growth model, line graph, recursive description, scatter plot

Students will be able to:
- Determine the number of pairs in the Nth generation of population growth.
- Apply a linear model to population growth, both recursively and explicitly.
- Use a line plot or scatter plot to represent population growth.
- Find the arithmetic sum for a specified number of terms.
- Apply an exponential model to population growth, both recursively and explicitly.
- Apply the compounding rule for annual compounding and general compounding.
- Find the geometric sum for a specified number of terms.

Resources

Core Text:
Excursions in Modern Mathematics. Peter Tannenbaum. Pearson Education, 2004. *Chapter 10*
Suggested Resources:
Excel, Google Sheets, Desmos

UNIT 3: Statistics and Probability

Summary and Rationale

This unit deals with how statistics affect our lives through government policies, insurance rates, our health and diet, and public opinion. Students will build upon their understanding of the basic aspects of statistics and how data should be collected. It will also extend an understanding of how data can be summarized so that it is intelligible and measures the inherent uncertainty. Students will explore how to draw meaningful conclusions and use statistics to predict patterns in future events.

Recommended Pacing

18-20 blocks

State Standards

Standard S-ID Interpret Categorical and Quantitative Data

CPI #	Cumulative Progress Indicator (CPI)
1	Represent data with plots on the real number line (dot plots, histograms, and box plots).
2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Standard S-IC Making Inferences and Justifying Conclusions

CPI #	Cumulative Progress Indicator (CPI)
1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population
2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6	Evaluate reports based on data.

Standard S-CP

CPI #	Cumulative Progress Indicator (CPI)
1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Instructional Focus

Unit Enduring Understandings

- Change occurs when understanding is applied.
- Patterns can help to bring clarity and understanding.
- Models derived from date shape decision-making.

Unit Essential Questions

- How is mathematics used to measure, model, and calculate change?
- What are the patterns in the information we collect and how are they useful?
- How can mathematics be used to provide models that help us interpret data and make predictions?
- In what ways can data be expressed so that its accurate meaning is concisely presented to a specific audience?

Objectives

Students will know:

- Ways to describe, summarize and interpret data sets.
- The difference between qualitative and quantitative data and continuous or discrete variables.
- The numerical summaries of data sets and what they can reveal about the data.
- A good profile can be developed for a data set using the Five-Number Summary.
- The standard deviation is the deviation from the mean.
- The sample space is the set of all possible outcomes for a random experiment.
- Order matters in permutations and order does not matter with combinations.
- Probability is the likelihood of an event happening.
- The difference between theoretical and experimental probabilities.
- The difference between probability and odds of an event happening.

Vocabulary:

average, bar graph, box plot, categorical variable, category, class interval, continuous variable, data set, data value, deviation from mean, discrete variable, five-number summary, frequency, frequency table, histogram, interquartile range, locator, measures of location, certain event, combination, even, impossible event, independent event, multiplication rule, odds, permutations, probability assignments, probability space, random experiment, sample space

Students will be able to:

- Create graphical representations of data sets, such as frequency tables, bar graphs, and pictograms and identify any outliers for the data set.
- Read and interpret the different types of data sets which use categorical representations, such as pie charts and histograms.

- Calculate and interpret the measures of location (mean, median, and quartiles) and measures of spread (range, interquartile range, and standard deviation).
- Calculate percentiles to sort data sets.
- Create box plots and calculate the five-number summary.
- Calculate standard deviation using the variance.
- Describe the sample space for a random experiment for single events and multiple events, using the multiplication rule.
- Calculate the number of permutations or combinations for a random event.
- Calculate the probability of single and multiple events happening when all outcomes are equally likely.
- Calculate the odds of an event occurring or against an event occurring.

Resources

Core Text:
Excursions in Modern Mathematics. Peter Tannenbaum. Pearson Education, 2004. *Chapters 14 and 15*

Suggested Resources:
Excel, Google Sheets, Desmos

UNIT 4: The Mathematics of Social Choice

Summary and Rationale

This unit deals with mathematical applications in social science. Students will learn how decisions are made, how elections are decided, how fairness is determined, and how other social choice issues are mathematically determined. Students will gain insight into how election models vary from place to place and their limitations in calculation methods

Recommended Pacing

15-20 blocks

State Standards

Standard S-MD Using Probability to Make Decisions

CPI #	Cumulative Progress Indicator (CPI)
6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Instructional Focus

Unit Enduring Understandings

- Mathematics and the models associated help to make sense of how real-life decisions can be made.

Unit Essential Questions

- How is mathematics used to quantify and compare situations, events and phenomena?

Objectives

Students will know:

- The terminology involved with social choice
- There are different methods to determine the winner of a vote.
- One method may be better to use than another depending on the situation.
- There are different voting methods that will give one person more power than another
- Whether a voting method is fair
- The ranking of the vote is important
- Understand the problems involved with each voting method
- Understand that people can have more voting power than others
- Understand the electoral college, its advantages, and disadvantages.
- Vocabulary associated with fair division.

Vocabulary:

Arrow's impossibility theorem, Borda count method, extended rankings methods, insincere voting, linear ballot, majority criterion, plurality method, preference ballot, preference schedule, rankings, weighted voting system, multiplication rule, motion, losing coalition, grand collision, coalition weight, continuous fair-division game, discrete fair-division game, divider-chooser method, fair division, fair-division method, fair share, last-diminisher method, lone-chooser method, lone-divider method, method of markers, methods of sealed bids.

Students will be able to:

- Collect data and create a preference schedule from ballets.
- Be able to determine the winner or rank the winners in the 4 different types of voting methods.
- Determine if someone has full or partial control of a vote.
- Mathematically determine the percentage of power a person has in a vote.
- Explain the electoral college, and compare and contrast it to other voting methods to determine if it is most beneficial or fair.
- Determining the term and percentages of fair division.

Resources

Core Text:

Excursions in Modern Mathematics. Peter Tannenbaum. Pearson Education, 2004. *Chapters 1, 2, 3*

Suggested Resources:

Excel, Google Sheets, Desmos