PISCATAWAY
TOWNSHIP SCHOOLS

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College Algebra

Content Area: Mathematics

Grade Span: 11-12

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COURSE OVERVIEW

Description
College Algebra is offered in cooperation with Middlesex County College. Three credits in MAT116 - College Algebra at Middlesex County College may be earned simultaneously with 5 credits earned at Piscataway High School. These credits are transferable to other institutions. This course is designed to prepare students for general education science and mathematics electives at college. Topics include concepts of algebra, algebraic functions and graphs, exponential and logarithmic functions and graphs, inequalities, and systems of equations. Applications are emphasized. The course will focus on writing in mathematics, problem-solving, and modeling. The content of this course will be aligned with the Common Core Standards for Mathematics where appropriate. To receive credit from Middlesex County College, students are required to maintain at least a 70 average for the course and must pay a one-time registration fee to the college.

Goals
In addition to the content standards, skills, and concepts set forth, this course also seeks to meet the Standards for Mathematical Practice. These practices include generally applied best practices for learning mathematics, such as understanding the nature of proof and having a productive disposition towards the subject, and are not tied to a particular set of content. These skills are applicable beyond a student's study of mathematics.   The eight Standards for Mathematical Practice are outlined below: 1. Make sense of problems and persevere in solving them.   2. Reason abstractly and quantitatively.   3. Construct viable arguments and critique the reasoning of others.   4. Model with mathematics.   5. Use appropriate tools strategically.   6. Attend to precision.   7. Look for and make use of structure.   8. Look for and express regularity in repeated reasoning.

Scope and Sequence

Unit	Topic	Length (90 Blocks)
1	Graphing	12
2	Systems of Equations, Inequalities	14
3	Piecewise and Absolute Value Functions	12
4	The Algebra of Functions	14
5	Quadratics and Rational Functions	12
6	Trigonometry Review	8
7	Exponential and Logarithmic Functions	12
8	Senior Project	6

Resources

Core Text: Larson, Ron. *College Algebra*. Brooks/Cole, 2021.

Suggested Resources: Delta Math, IXL Learning, Kuta – Algebra 2, Delta Math, Desmos

UNIT 1: Graphing

<table>
<tr><td colspan="2" align="center">**Summary and Rationale**</td></tr>
<tr><td colspan="2">This introductory unit reviews Algebra 2 concepts and understanding of graphing linear functions. Extensive comparison of vertical, horizontal, and slanted linear graphs as it relates to their slopes and intercepts. Students also explore scatter plots, parallel and perpendicular graphs, and their slope relationships. And writing linear graphs given slope and ordered-pair point on the line. Modeling and problem-solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.</td></tr>
<tr><td colspan="2" align="center">**Recommended Pacing**</td></tr>
<tr><td colspan="2" align="center">12 Blocks</td></tr>
<tr><td colspan="2" align="center">**State Standards**</td></tr>
<tr><td colspan="2">**Standard S-ID Interpreting Categorical and Quantitative Data**</td></tr>
<tr><td>**CPI #**</td><td>**Cumulative Progress Indicator (CPI)**</td></tr>
<tr><td>6b</td><td>Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.</td></tr>
<tr><td>6c</td><td>Fit a linear function for a scatter plot that suggests a linear association.</td></tr>
<tr><td>7</td><td>Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</td></tr>
<tr><td>9</td><td>Distinguish between correlation and causation.</td></tr>
<tr><td colspan="2">**Standard F-IF Interpreting Functions**</td></tr>
<tr><td>**CPI #**</td><td>**Cumulative Progress Indicator (CPI)**</td></tr>
<tr><td>7a</td><td>Graph linear and quadratic functions and show intercepts, maxima, and minima.</td></tr>
<tr><td colspan="2" align="center">**Instructional Focus**</td></tr>
<tr><td colspan="2">**Unit Enduring Understandings**</td></tr>
<tr><td colspan="2">

- Know how to represent a real-world linear relationship.
- The best representation of a linear relationship is based on the data available.
- There are mathematical patterns everywhere.
- Changing one parameter of a function will change the look of the image.
- The zeroes of a function are the roots or x-intercepts

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<tr><td colspan="2">**Unit Essential Questions**</td></tr>
<tr><td colspan="2">

- How can one use graphs and equations to identify linear functions?
- What are intercepts and how can they be used?
- How can one relate the rate of change and slope in linear relationships?
- How is the rate of change in a real-world linear relationship related to the slope-intercept form of the equation that represents the relationship?
- What is the standard form of a linear equation? If an equation is in standard form, what are the exponents of x and y?
- What do you notice about the y-coordinates of the points on a horizontal line? What do you notice about the x-coordinates of the points on a vertical line?
- How can you identify and use intercepts in linear relationships?
- What are the intercepts of the line with equation y = x? Describe the graph of the line with the equation y = x. Do the intercepts make sense?
- Is it necessary to write an equation in standard form?
- Does it matter which point you start with to find the slope? What does a line with a positive slope look like? negative slope? zero slope?

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• Why does a line not always go through the origin?

Objectives

Students will know:
- The standard form of a linear equation is Ax + By = C, where A, B, and C are real numbers and A and B, are not both 0.
- The graphs of linear functions are either vertical, horizontal, or slanted-line.
- The equation of a horizontal line is of the form y = k. The equation of a vertical line is of the form x = k.
- The slope-intercept form of a linear equation is y = mx + b, where m represents the slope of the line described by the equation, and b represents the y-intercept.
- A linear function can be graphed by plotting the y-intercept and using the slope to find other points that lie on the line.
- The slope-intercept form of a linear equation can be used to write functions that model real-world situations.
- There are multiple ways to represent a function, and being able to convert from one form to another is helpful when comparing functions.
- It is often useful to convert functions to the same format in order to compare them.
- The slope of parallel and perpendicular graphs.

Vocabulary: equation in two variables, solution or solution point, ordered-pair points, intercepts, slope, symmetry, scatter plot, correlation, parallel, perpendicular, undefined slope, zero slopes, regression line

Students will be able to:
- Practice creating and graphing linear equations and apply these skills to the rest of the unit.
- Represent real-life applications by creating scatter plots.
- Analyze the differences between parallel and perpendicular linear graphs.
- Analyze the type of function represented.
- Identify the dependent and independent variables.
- Identify x-intercepts and y-intercepts of graphs.
- Create and analyze lines of best fit for scatter plots.
- Make predictions based on the data used in real-life applications.
- Create the line of best fit based on a set of data.
- Identify the slope and function of both vertical line graphs.

Resources

Core Text: Larson, Ron. *College Algebra*. Brooks/Cole, 2021.

Suggested Resources: Delta Math, IXL Learning, Kuta – Algebra 2,Delta Math, Desmos

UNIT 2: Systems of Equations, Inequalities

Summary and Rationale
This unit builds from Algebra 2 knowledge and skills. The unit involves a brief review of solving equations and Inequalities. Students use their abilities to solve equations to Solve Systems of Equations in two variables by the following methods: graphing calculators; substitution; linear combination. Furthermore, solving 3-variable system equations and graphing systems of Linear Inequalities is covered in Unit 2. Modeling and problem-solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

Recommended Pacing
14 Blocks

State Standards

Standard A-REI Reasoning with Equations and Inequalities

CPI #	Cumulative Progress Indicator (CPI)
3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions
6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
12	Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Standard A-CED Creating Equations

CPI #	Cumulative Progress Indicator (CPI)
1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Instructional Focus

Unit Enduring Understandings

- Know how to represent a real-world linear relationship.
- The best representation of a linear relationship is based on the data available.
- The symbolic language of algebra is used to communicate and generalize the patterns in mathematics
- Algebraic representation can be used to generalize patterns and relationships
- Patterns and relationships can be represented graphically, numerically, symbolically, or verbally
- Mathematical models can be used to describe and quantify physical relationships
- Physical models can be used to clarify mathematical relationships
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole
- Reasoning and/or proof can be used to verify or refute conjectures or theorems in algebra

Unit Essential Questions

- Is Mathematics a language?
- What is the most effective way to solve a problem?
- Why are equations and inequalities useful?
- What makes a relationship linear?
- How many solutions are there for systems of equations?
- How many solutions are there for inequalities?
- Why does the solution set look different based on the type of system?
- How do you know if there is no solution?
- How does a compound inequality graph look different from a simple inequality graph?
- How do you know if the circle is open or closed?
- Which is the best way to solve a system of equations?
- What words in an application problem help you determine the type of sign that you should use?
- How do you know the solution region to the system of inequalities?

Objectives

Students will know:
- How to solve a system of 2x2 and 3x3 linear equations.
- A system of equations is a set of more than one equation (although one can be an equivalent equation).
- Systems can have no solution, 1 solution, or infinite solutions.
- The elimination (or linear combination) method eliminates one of the variables by creating opposites.
- Substitution can be solved using 2 methods.
- How to solve using graphing or using the graphing calculator.
- There are many applications of systems of equations.
- How to solve a system of linear inequalities.

Vocabulary: systems of equations, solution, infinite solutions, no solution, substitution, elimination, linear combination, points of intersection, graphical method, linear, non-linear, Gaussian elimination, solution region, shaded region, greater than or less than, greater or equal, less than or equal, solution set, interval notation

Students will be able to:
- Calculate and determine the solution set for systems of equations.
- Graph systems using a graphing calculator and find the point of intersection.
- Solve a system of 2x2 linear equations numerically and graphically.
- Solve a system of 2x2 linear equations using the substitution and elimination (linear combination) method.
- Solve a system of 3x3 linear equations
- Solve an equation of the form $ax+b=cx+d$ for x.
- Interpret a real-world problem and create the equations that it represents. Then students will use various methods to find the solution set including the graphing calculator.
- Solve inequalities and write their solution interval.
- Solve a system of inequalities and determine its solution region.
- Solve linear inequalities in one variable numerically and graphically.
- Use properties of inequalities to solve linear inequalities in one variable algebraically.
- Solve compound inequalities algebraically.
- Use interval notation to represent a set of real numbers described by an inequality.
- Graph the solution set for inequalities.

Resources

Core Text: Larson, Ron. *College Algebra*. Brooks/Cole, 2021.
Suggested Resources: Delta Math, IXL Learning, Kuta – Algebra 2, Delta Math, Desmos

UNIT 3:   Piecewise and Absolute Value Functions

Summary and Rationale
The Unit is based on understanding from Algebra 2 and its focus on functions and transformations. Piecewise Functions will be reintroduced as part of the set of linear, exponential, and Absolute Value graphs and will be explored in a number of equation forms. Absolute value functions, piecewise functions, and linear inequalities will also be explored. Explorations in this unit set the standard for the graph analysis that will occur during the course, including analysis of the meaning of a solution, an understanding of domain and range, and the effects of transformations on each function explored. Modeling and problem-solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

Recommended Pacing
12 Blocks

State Standards

Standard F-IF Interpreting Functions

CPI #	Cumulative Progress Indicator (CPI)
1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Standard A-REI Reasoning with Equations and Inequalities

CPI #	Cumulative Progress Indicator (CPI)
11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions

Instructional Focus

Unit Enduring Understandings

- Know how to represent a real-world linear relationship.
- The best representation of a linear relationship is based on the data available.
- Piecewise functions can be used to represent real-life applications.
- All functions are not linear.
- The symbolic language of algebra is used to communicate and generalize the patterns in mathematics
- Algebraic representation can be used to generalize patterns and relationships
- Patterns and relationships can be represented graphically, numerically, symbolically, or verbally

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- Mathematical models can be used to describe and quantify physical relationships
- Physical models can be used to clarify mathematical relationships
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole
- Reasoning and/or proof can be used to verify or refute conjectures or theorems in algebra

Unit Essential Questions

- Is mathematics a language?
- What is the most effective way to solve a problem?
- Why are equations and inequalities useful?
- What makes a relationship linear? Non-linear?
- How can change be best represented mathematically?
- How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?
- How are patterns of change related to the behavior of functions?
- How can we use mathematical models to describe physical relationships?
- How can we use physical models to clarify mathematical relationships?
- What makes an algebraic algorithm both effective and efficient?
- Compared to other functions, what are the similarities and differences to Piecewise Functions?

Objectives

Students will know:
- The meaning of a piecewise function
- The form of an equation for a horizontal line
- The form of an equation for a vertical line
- Real-life applications for these types of equations
- Graph Properties of Absolute Value Functions
- Parent and Transformations of Absolute Value Functions

Vocabulary: piecewise function, continuity, inequalities, greater than or less than, greater or equal, less than or equal, absolute value, parent graph, transformation

Students will be able to:
- Graph a piecewise linear function
- Write a piecewise linear function to represent a given situation
- Determine the equation of a horizontal line.
- Determine the equation of a vertical line.
- Interpret a real-world problem and create the equations that it represents.
- Identify the shape of a graph given the equation of a function
- Graph the set of solutions to an equation, an inequality, an absolute value function, a piecewise function, and a circle
- Graph analysis of linear, absolute value, piecewise, linear inequalities, and circles
- Identify the domain and range of a given function using set builder and interval notation
- Identify the input and output of a function in proper function notation
- Compare $f(x)$ to $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for absolute value functions
- Evaluate an output for a function given a specific input
- Identify a type of relation given a table of values
- Review solving absolute value equations, inequalities, special cases (i.e, x<2 and x<3, x<2 or x<3)
- Rewrite an absolute value inequality into two separate single-variable cases
- Use set-builder notation and interval notation to express solutions

Resources
Core Text: Larson, Ron. *College Algebra*. Brooks/Cole, 2021.   **Suggested Resources:** Delta Math, IXL Learning, Kuta – Algebra 2, Delta Math, Desmos

UNIT 4: The Algebra of Functions

Summary and Rationale
The Algebra of Functions Unit takes a detailed study of Functions starting with an introduction to function definition and function notation. The unit includes analyzing graphs of functions, linear and nonlinear Parent functions, and their transformations. Furthermore, the unit covers a combination of functions, composite functions, and inverse functions.   Modeling and problem-solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

Recommended Pacing
14 Blocks

State Standards

Standard F-IF Interpreting Functions

CPI #	Cumulative Progress Indicator (CPI)
1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Standard A-APR Arithmetic with Polynomials and Rational Expressions

CPI #	Cumulative Progress Indicator (CPI)
1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials

Standard F-BF Building Functions

CPI #	Cumulative Progress Indicator (CPI)
3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
4	Find inverse functions.
4a	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or f(x) = (x+1)/(x−1) for x ≠1
4b	Verify by composition that one function is the inverse of another.
4c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
4d	Produce an invertible function from a non-invertible function by restricting the domain.

Instructional Focus

Unit Enduring Understandings

- The symbolic language of algebra is used to communicate and generalize the patterns in mathematics
- Algebraic representation can be used to generalize patterns and relationships
- Patterns and relationships can be represented graphically, numerically, symbolically, or verbally
- Mathematical models can be used to describe and quantify physical relationships
- Physical models can be used to clarify mathematical relationships
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole
- Reasoning and/or proof can be used to verify or refute conjectures or theorems in algebra
- Know how to represent a real-world linear relationship.
- The best representation of a linear relationship based on the data available.
- There are mathematical patterns everywhere.
- To change one parameter of a function will change the look of the image.
- The zeroes of a function are the roots or x-intercepts

Unit Essential Questions

- What makes a graph a function?
- When is a graph not a function?
- Can a graph be both a function and one-to-one?
- What is the domain and range of functions?
- How is Composition Function related to Inverse Function
- How do Parent Functions and their Transformations connect to linear and nonlinear Function graphs?

Objectives

Students will know:
- Domain and Range of Functions
- Function Notation
- Linear and Nonlinear graph Functions and their Transformations
- Composition Function and its connection to Inverse Function
- Combination of Functions changes the Function
- Real-life Applications about linear and nonlinear Functions

Vocabulary: domain, range, inclusive, exclusive, intersection, union, compound inequality, absolute value, x-intercept, y-intercept, set-builder notation, interval notation, axis of symmetry, piecewise function, vertical/horizontal stretch/compression/shift, inverse function, combination of function, composition function

Students will be able to:
- Determine whether relations between two variables are functions.
- Find the domain and range of functions.
- Use functions to model and solve real-life problems, and evaluate different quotients.
- Use the Vertical Line Test for Functions.
- Find the Zeros of Functions.
- Determine intervals of which Functions are increasing or decreasing.
- Determine the relative minimum and relative maximum value of functions.
- Recognize graphs of commonly used parent functions.
- Use vertical and horizontal shifts to sketch graphs of functions.
- Use reflection to sketch graphs of functions.
- Use nonrigid transformations to sketch graphs of functions.
- Add, subtract, multiply, and divide functions.
- Find the composition of one function with another function.
- Use combination and composition of functions to model and solve real-life problems.

Resources
Core Text: Larson, Ron. *College Algebra*. Brooks/Cole, 2021. **Suggested Resources:** Delta Math , IXL Learning, Kuta – Algebra 2, Delta Math, Desmos

UNIT 5: Quadratic and Rational Functions

Summary and Rationale
This unit continues Algebra 2 understanding of quadratic functions, their transformations within the context of, and Rational Functions. An extensive comparison of the three quadratic forms and their solutions will elucidate the algebraic relationships between the forms and will provide an understanding of the value of each equation in graph analysis. Students will also be introduced to the set of complex numbers and will explore the relationship between quadratic graphs and complex solutions. This unit will also include basic introductions of end behavior, rate of change, and limits. Modeling and problem-solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

Recommended Pacing
12 Blocks

State Standards

Standard F-IF The Complex Number System

CPI #	Cumulative Progress Indicator (CPI)
7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Standard N-CN The Complex Number System

CPI #	Cumulative Progress Indicator (CPI)
1	Know there is a complex number i such that i^2 = –1, and every complex number has the form a + bi with a and b real.
2	Use the relation i^2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers
7	Solve quadratic equations with real coefficients that have complex solutions

Standard A-SSE Seeing Structure in Expressions

CPI #	Cumulative Progress Indicator (CPI)
3a	Factor a quadratic expression to reveal the zeros of the function it defines.

Standard A-SSE Seeing Structure in Expressions

CPI #	Cumulative Progress Indicator (CPI)
4	Solve quadratic equations in one variable.
4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
4b	Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Instructional Focus

Unit Enduring Understandings

- There are mathematical patterns everywhere
- The zeroes of a function are the roots or x-intercepts
- To change one parameter of a function will change the look of the image.
- Rational Functions have asymptotes

- There is more than one way to find the roots of a quadratic function
- Is an equation defined for all values of x
- Boundaries must be clear for good decision making
- Analyzing trends predicts behavior
- Real-life scenarios can be modeled with quadratic equations.

Unit Essential Questions

- What are the critical attributes of a given function?
- How do you transform a graph
- How can you restrict a function?
- What will you expect a family of functions to look like?
- Howis the x-intercepts of a quadratic function related to the solutions of a quadratic equation?
- How many ways can you find the roots of a quadratic function?
- What does y approach as x approaches infinity or negative infinity?
- Can you really divide fractions?
- How do you find excluded values of a fraction?
- Can understanding a relationship help find a pattern?
- How do the parts affect the whole?

Objectives

Students will know:

- How to write an equation of a function given critical attributes of the function
- How to write an equation of a function given critical attributes of the function.
- An algorithm for factoring quadratic functions.
- The quadratic formula can be derived by completing the square on a standard form quadratic equation
- The definition of i and the pattern for the powers of i
- A complex number is the sum of a real part and an imaginary part. (a+bi)
- Complex conjugate solutions to quadratic equations always come in pairs.
- A complex solution to a quadratic equation implies there is no x-intercept to the related quadratic function.
- A quadratic function can be written in vertex form, standard form, and intercept form.
- How to find critical attributes of a parabola given a specific form of a quadratic function.
- The y-intercept occurs when x = 0 and the x-intercept occurs when y= 0.
- The roots (solutions) to a quadratic equation are the zeros (x-intercepts) of the related quadratic function.
- For all even functions f(-x) = f(x) and there is always y-axis symmetry
- Quadratic functions are never odd functions and are only even if the graph is limited to a vertical translation
- Basic shapes of linear, absolute value, and quadratic parent graphs as well as the size of circles centered at the origin
- Proper notation for domain, range, and end-behavior of rational models
- Asymptotes of rational functions are related to domain and range
- End behaviors of rational functions are related to limits as x approaches infinity or negative infinity
- A continuous function can be drawn without lifting your pencil
- Vertical asymptotes affect the interval of continuity for a rational function
- Dividing two rational expressions is equivalent to multiplying by the reciprocal of the divisor
- When a common factored is reduced, that assumed vertical asymptote of the related function becomes a hole
- (point of discontinuity) Basic shapes of linear, absolute value, quadratic, power, trigonometric, exponential, and rational parent functions

Vocabulary: domain,range,inclusive, exclusive, *x*-intercept (zeros), *y*-intercept, set-builder notation,interval notation, axis of symmetry, quadratic functions,vertex,complex number system,rational, irrational, imaginary

number, quadratic formula, discriminant, completing the square, factor, roots, zeros, vertical/horizontal stretch/compression/shift, Continuity, Asymptote, rational function, end behavior, point of discontinuity (hole), complex fraction, extraneous solution, direct, inverse, and joint variation

Students will be able to:

- Create equations to represent relationships and to solve real world problems
- Factor to find roots of quadratic functions
- Identify difference of squares and perfect square trinomials

- Use quadratic techniques to connect (x +4i)(x −4i) and $x^2 + 16$
- Define i and simplify powers of i
- Connect complex solutions of quadratic equations to imaginary zeroes of quadratic functions (through
- quadratic formula and completing the square)
- Solve the same quadratic equation four ways: factoring, quadratic formula, completing the square, looking at
- the related graph
- Identify critical attributes of a graph from the three forms of a quadratic function
- Transition from one form of a quadratic to another
- Find x- and y-intercepts of graphs
- Write equations of quadratics given a vertex and a point on the curve
- Graph systems of quadratics, absolute value, and linear functions

Resources

Core Text: Larson, Ron. *College Algebra*. Brooks/Cole, 2021.

Suggested Resources: Delta Math ,IXL Learning, Kuta – Algebra 2, Delta Math, Desmos

UNIT 6: Trigonometry Review

	Summary and Rationale

In this Unit, the focus is on trigonometric review concepts about right triangle trigonometry. The unit addresses the Pythagorean Theorem, solving special right triangles (30-60-90 and 45-45-90), and finding a side when only one side is given. Furthermore, students will evaluate the six trigonometric ratios using the right triangle and using a calculator and finding angles when two sides of a right triangle are given. The angle of elevation and depression (indirect measurement) will be introduced directly through real-world applications. Modeling and problem-solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

Recommended Pacing
8 Blocks

State Standards

Standard G Geometry

CPI #	Cumulative Progress Indicator (CPI)
5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Standard G-SRT Similarity, Right Triangles, and Trigonometry

CPI #	Cumulative Progress Indicator (CPI)
6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7	Explain and use the relationship between the sine and cosine of complementary angles.
8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
9	(+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10	(+) Prove the Laws of Sines and Cosines and use them to solve problems
11	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Standard F-TF Trigonometric Functions

CPI #	Cumulative Progress Indicator (CPI)
1	Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3	(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number.
4	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
6	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

7	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Instructional Focus

Unit Enduring Understandings

- Analyzing trends predict behavior.
- There are many ways to solve a problem.
- Relationships vary by situation.
- Changing one or many pieces can change the whole.
- Characteristics allow you to make predictions.
- Multiple models exist for a single scenario.
- Procedures can be reversed.
- There are six trigonometric ratio functions related to a right triangle.
- Trig functions allow you to solve any right triangle
- Real-world phenomena can be modeled using trig functions.

Unit Essential Questions

- Why are predictions beneficial?
- When there are multiple approaches, how should you choose the best method?
- Can understanding a relationship help make a prediction?
- How do the parts affect the whole?
- How do your characteristics relate you to your family?
- Is there a need for different viewpoints?
- What is the effect of interchanging parts?
- How can you use trig ratios to solve problems that model triangles?
- How do you evaluate the six trig functions for any given reference angle measure without a calculator?

Objectives

Students will know:
- The relationship between the sides of special 30-60-90 and 45-45-90 right triangles
- There are six trigonometric ratio functions related to a right triangle
- Trig functions allow you to solve any right triangle
- The significance of coordinates along the unit circle, and how they relate to the six trig functions
- There are two units of angle measurements: degrees and radians
- The radian measure of an angle is the length of the arc on the unit circle
- The equation of the unit circle
- On the unit circle, x = cos(A) and y= sin(B)
- Real world phenomena can be modeled using trig functions
- The Pythagorean identity $sin^2 x + cos^2 x = 1$
- The measure of an angle in degrees or radians is positive if its rotation is counterclockwise from the positive x-axis.
- End behavior of oscillating graphs is undefined since periodic functions will not converge to a single value
- Proper notation for domain and range of trigonometric functions
- One complete rotation is 360 degrees or 2pi radians
- The value of a trigonometric ratio is dependent on what quadrant the angle's terminal side is in
- The period does not influence range but the amplitude and vertical shift do
- For all even functions f(-x) = f(x) and there is always y-axis symmetry
- For all odd functions f(-x) = -f(x) and there is always 180-degree rotational symmetry about the origin
- Some functions are neither odd nor even
- Basic shapes of linear, absolute value, quadratic, power, and trigonometric parent functions as well as circles
- Differentiating attributes of all functions graphed so far

Vocabulary: Six Trigonometric Functions: Sine, Cosine, Tangent, Secant, Cosecant, Cotangent, Reference Angle, Quadrantal Angles, Standard Position, X-axis, positive angle (counterclockwise), negative angle (clockwise), Coterminal angles, Asymptote, oscillating curve, period, amplitude, Pythagorean theorem, 30-60-90 and 45-45-90 special right triangles

Students will be able to:
- Use the six trig functions to solve right triangles
- Given the value of one trigonometric function in a specific quadrant, find the value of the other five
- Convert between radians and degrees
- Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$
- Graph the parent functions $f(x) = \sin(x)$, $f(x) = \cos(x)$, and $f(x) = \tan(x)$
- Compare $f(x)$ to $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for trigonometric functions
- Reproduce coordinates along the unit circle using knowledge of reference angles and symmetry
- Evaluate six trig functions for any given angle measure without a calculator
- Find a reference angle and a coterminal angle and know how they are related
- Model real world phenomena using trig functions
- Identify the amplitude, period, phase shift, and vertical shift for sine and cosine
- Graph analysis of sine, cosine, and tangent functions
- Examine domain and range of all functions covered
- Solve trigonometric functions using quadratic techniques
- Estimate the measure of an angle in radians, some examples not drawn in standard position
- Use a graphing calculator "window" in radians to graph 2 periods of trigonometric functions

Resources

Core Text: Larson, Ron. *College Algebra*. Brooks/Cole, 2021.

Suggested Resources: Delta Math , IXL Learning, Kuta – Algebra 2, Delta Math, Desmos

UNIT 7:   Exponential and Logarithmic Function

Summary and Rationale
In this unit students will apply their understanding of inverse functions to exponential functions in order to explore the concept of the logarithm. Algebraic analysis of the logarithm and its components will allow students to understand its purpose within the course and the construction of its graph. Continued comparison with exponential functions will explore the nature of asymptotes, the shape of graphs, and transformations on the resulting function. Students will explore a number of logarithmic bases and will be introduced to the natural log and its connection with Euler's number. Modeling and problem-solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

Recommended Pacing
12 Blocks

State Standards

Standard A-CED Creating Equations

CPI #	Cumulative Progress Indicator (CPI)
4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Standard A-REI Reasoning with Equations and Inequalities

CPI #	Cumulative Progress Indicator (CPI)
11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are exponential functions.

Standard A-SSE Reasoning with Equations and Inequalities

CPI #	Cumulative Progress Indicator (CPI)
1	Interpret expressions that represent a quantity in terms of its context.
1a	Interpret parts of an expression, such as terms, factors, and coefficients.
2	Use the structure of an expression to identify ways to rewrite it.
3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
3c	Use the properties of exponents to transform expressions for exponential functions.

Standard F-BF Building Functions

CPI #	Cumulative Progress Indicator (CPI)
4c	(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

Standard F-IF Reasoning with Equations and Inequalities

CPI #	Cumulative Progress Indicator (CPI)
7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

8b	Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Standard F-LE Linear and Exponential Models

CPI #	Cumulative Progress Indicator (CPI)
1	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standard N-Q Quantities

CPI #	Cumulative Progress Indicator (CPI)
1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2	Define appropriate quantities for the purpose of descriptive modeling

Instructional Focus

Unit Enduring Understandings

- Analyzing trends predict behavior.
- There are many ways to solve a problem.
- Relationships vary by situation.
- Changing one or many pieces can change the whole.
- Characteristics allow you to make predictions.
- Multiple models exist for a single scenario.
- Procedures can be reversed.
- Exact answers are not always the best answers.
- Boundaries must be clear for good decision-making.
- Relationships show connections.
- Appreciation and depreciation affect the rate of growth.

Unit Essential Questions

- Why are predictions beneficial?
- When there are multiple approaches, how should you choose the best method?
- Can understanding a relationship help make a prediction?
- How do the parts affect the whole?
- How does your characteristics relate you to your family?
- Is there a need for different viewpoints?
- What is the effect of interchanging parts?
- Can there be more than one correct solution?
- When is the correct answer not the best solution?

Objectives

Students will know:
- Exponential functions are used to model problems involving banking, growth (doubling) and decay (half-life)

- Proper notation for domain, range and end-behavior of exponential models
- Exponential models can roughly fit data and be used for predictions
- Exponential functions have a horizontal asymptote
- The range of an exponential model can only approach an asymptote and should be expressed as an inequality
- The limit as x approaches negative infinity for an growth model is the asymptote of the graph
- The limit as x approaches infinity for a decay model is the asymptote of the graph
- Basic shapes of linear, absolute value, quadratic, power, trigonometric and exponential parent functions
- Differentiating attributes of all functions graphed so far
- Euler's number (e) is a very important irrational number used in mathematics approximated by 2.718
- $y=ab^x$ is the basic model for exponential functions where a is the initial value and b is the growth factor
- Exponential growth and decay formulas: $A= P(1 \pm r)t$ where r is the growth rate and $A=Pe^{rt}$
- Banking compound interest formula: $A=P(1+r/n)nt$ and compound continuously formula: $A=Pe^{rt}$
- Basic properties of logs:
    1) $\log 1 = 0$; $\ln 1 = 0$
    2) $\log_b b =1$; $\ln e = 1$
    3) $\log a^x = x\log a$; $\ln e^x =x$
- Logarithmic functions have a vertical asymptote
- A logarithm can be used to find the value of an exponent to a power function.
- The domain of f(x) = log x only includes positive values and the range includes all real numbers.
- Logarithmic functions and exponential functions are inverses of each other.
- The limit as x approaches infinity for the basic logarithm parent graphs is infinity
- natural log means log base e
- Common log means log base 10
- A logarithmic function always has a vertical asymptote
- Proper notation for domain, range and end-behavior of logarithmic model
- Logarithmic functions are never continuous on the interval from negative infinity to positive infinity

Vocabulary: domain, range, end-behavior, exponential growth, exponential decay, geometric, arithmetic, asymptote, limit, base, power, growth rate, growth factor, doubling (b = 2), tripling (b = 3), half-life, e, compound interest, values of n for annually, semiannually, quarterly, monthly, daily and compounded continuously, Exponential functions, Logarithmic functions, base, power, change of base formula, limit, Inverse, continuity

Students will be able to:
- Derive exponential growth and decay equations that model real-world situations.
- Solve exponential equations by renaming the base.
- Sketch a graph of an exponential function and use proper notation to state the function's domain and range.
- Write an exponential function given the y-intercept and an additional solution to the function.
- Identify asymptotes and intercepts of exponential equations before graphing.
- Graph analysis of exponential functions
- Use exponential modeling for growth (doubling), decay (half-life), and compound interest
- Compare f(x) to f(x)+k, kf(x), f(kx), and f(x+k) for exponential functions
- Use proper notation to find domain and range and end-behavior of exponential functions
- Connect exponential equations to geometric sequences
- Basic shapes of linear, absolute value, quadratic, power, trigonometric and exponential parent functions
- Differentiating attributes of all functions graphed so far

- Use a horizontal asymptote to connect to restrictions on the range of an exponential function
- Use the "calc-intersect" function on a graphing calculator to find break-even points
- Use the "tabl" and "tbl-set" function on a graphing calculator to find the value of time that would double or triple a real-world scenario

Resources

Core Text: Larson, Ron. *College Algebra*. Brooks/Cole, 2021.

Suggested Resources: Delta Math, IXL Learning, Kuta – Algebra 2, Delta Math, Desmos

UNIT 8:   Senior Project

Summary and Rationale
In this final Unit, students will be assigned a Quadratic application problem for which they will create a via PowerPoint or Google Slide presentation describing variations of the function. They will be required to create the correct functions, solve, graph, and discuss how changing the function relates to their real-life problem. Students determine the maximum height and the time at which their object reaches various key heights. Moreover, finding how increasing and decreasing velocity affects the maximum height and the trajectory of the object reaching ground level. Modeling and problem-solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

Recommended Pacing
6 Blocks

State Standards

Standard A-CED Creating Equations

CPI #	Cumulative Progress Indicator (CPI)
1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Standard A-REI Reasoning with Equations and Inequalities

CPI #	Cumulative Progress Indicator (CPI)
4	Solve quadratic equations in one variable.

Standard F-IF Interpreting Functions

CPI #	Cumulative Progress Indicator (CPI)
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship
7b	Graph linear and quadratic functions and show intercepts, maxima, and minima.
8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context

Instructional Focus

Unit Enduring Understandings

- Know how to represent real-world relationships.
- The best representation of a relationship based on the data available.
- All functions are not linear.
- There are different representations for numbers.

Unit Essential Questions

- Is Mathematics a language?
- What is the most effective way to solve a problem?
- Why are equations useful?
- What is the relationship between quadratic functions and real-life applications?
- How do logs help you solve exponential functions?
- What is the relationship between quadratic functions and their graphs?
- How does Desmos help you see the effects of graphs and what does its explanation mean?
- What is a practical domain or range?

• How does velocity affect the object's path trajectory?

Objectives

Students will know:
- Explore the graphs of quadratic functions
- Explore the graphs of quadratic functions
- Apply quadratic functions to various object projectiles
- How to find minimum and maximum value of quadratic functions
- Successful and effective ways of giving a Presentation

Vocabulary: Maximum Value, vertex, quadratic formula, roots, intercepts, gravity, velocity, standard form, trajectory, projectile

Students will be able to:
- Explore the graphs of quadratic functions
- Explore the graphs of quadratic functions
- Apply quadratic functions to various object projectiles
- Find minimum and maximum value of quadratic functions
- Find when an object reaches a certain distance
- Give effective class presentations

Resources

Core Text: Larson, Ron. *College Algebra*. Brooks/Cole, 2021.

Suggested Resources: Delta Math ,IXL Learning, Kuta – Algebra 2, Delta Math, Desmos