PISCATAWAY TOWNSHIP SCHOOLS

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Precalculus

Content Area:	**Mathematics**
Grade Span:	**11-12**
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COURSE OVERVIEW

Description

Precalculus courses combine the study of Trigonometry, Elementary Functions, Analytic Geometry, and Math Analysis topics as preparation for calculus from an approach of graphical, algebraic, numerical, tabular representations. Topics typically include the study of complex numbers; polynomial, logarithmic, exponential, rational, right, trigonometric, and circular functions, and their relations, transcendental functions, trigonometric identities, and analytic geometry. This course includes a review of necessary skills from algebra, reinforces polynomial, rational, exponential and logarithmic functions, and gives the student an in-depth study of trigonometric functions and their applications.

Modern technology provides tools for supplementing the traditional focus on algebraic procedures, such as solving equations, with a more visual perspective, with graphs of equations displayed on a screen. Students can then focus on understanding the relationship and behavior of the function, in preparation for the advanced study of calculus. Students will develop the connections between the graphical, algebraic, numerical, and tabular representations of these functions.

Goals

In addition to the content standards, skills, and concepts set forth, this course also seeks to meet the Standards for Mathematical Practice. These practices include generally applied best practices for learning mathematics, such as understanding the nature of proof and having a productive disposition towards the subject, and are not tied to a particular set of content. These skills are applicable beyond a student's study of mathematics.

The eight Standards for Mathematical Practice are outlined below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Scope and Sequence

Unit	Topic	Length (90 Blocks)
Unit 1	Functions Overview	8
Unit 2	Polynomial, Powers & Rational Functions	10
Unit 3	Logs & Exponential Functions with Applications	10
Unit 4	Analysis of Transcendental Functions	8
Unit 5	Introduction to Trig (Geometric Approach)	9
Unit 6	Introduction to Trig (Analytical Approach)	9
Unit 7	Graphs of Trig Functions	10
Unit 8	Trig Identities, Equations, and Applications	10
Unit 9	Analytic Geometry*	10
Unit 10	Sequences and Series*	4

Resources
Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, numerical, Algebraic*. Pearson. **Suggested Resources:**Desmos, Geogebra, Albert.io

UNIT 1: Functions Overview

Summary and Rationale

Functions and graphs form the basis for understanding mathematics and applications. This unit introduces some of the elementary functions students will encounter in the course. Although the functions are probably familiar, the graphical, numerical, verbal, and analytical (Rule of Four) approach to their analysis may be new.

Recommended Pacing

8 days

State Standards

Standard F-IF Interpreting Functions

CPI #	Cumulative Progress Indicator (CPI)
1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Standard F-BF Building Functions

CPI #	Cumulative Progress Indicator (CPI)
1b	Combine standard function types using arithmetic operations.
1c	Compose functions.
3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Instructional Focus

Unit Enduring Understandings

- There is more than one way to find the roots of a function.
- Relations and Functions can be represented graphically, numerically, analytically, or verbally.

Unit Essential Questions

- When is it more appropriate to analyze a function algebraically? Graphically?
- How do you transform a graph?
- What will you expect a family of functions to look like?
- What attributes are necessary when visualizing a function?

Objectives

Students will know:

- Standard form of the equation of a circle and the alternate form which is explicitly written for y.

- Difference between a constant, a variable and a function.
- A function is increasing over an interval of its domain if, as the input values increase, the output values always increase. That is, for all *a* and *b* in the in the interval, if $a < b$ then $f(a) < f(b)$.
- A function is decreasing over an interval of its domain if, as the input values increase, the output values always decrease. That is, for all *a* and *b* in the in the interval, if $a < b$ then $f(a) > f(b)$.
- A continuous graph attains a local maximum when the graph changes from increasing to decreasing.
- A continuous graph attains a local minimum when the graph changes from decreasing to increasing.
- An even function is graphically symmetric over the line $x = 0$ and analytically has the property $f(-x) = f(x)$.
- An odd function is graphically symmetric about the point $(0, 0)$ and analytically has the property $f(-x) =- f(x)$.
- Implicitly defined inverses explicitly defined equations.
- The transformations of a graph in function notation, namely, $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$.
- **Vocabulary:** domain,range, even/odd/, extrema, increasing/decreasing, end behavior, implicit equations, explicit equations, Rule of Four, average rate of change.

Students will be able to:

- Solve an equation graphically by either finding the roots after setting the function equal to zero or by graphing both sides of the equation and locating the points of intersection.
- Sketch the graph of a circle.
- Graph piecewise functions.
- Sketch $f(x) = \pi$ or $g(x) = e$
- Identify all attributes given a continuous or discontinuous graph
    - Domain and range
    - Symmetry (even or odd)
    - Intercepts
    - End behavior
    - Local and absolute extrema (Note, the y-coordinates are the extreme values)
    - Estimate intervals of concave up/concave down
    - Intervals of increasing/decreasing
    - Intervals of $f(x) > 0$ and $f(x) < 0$
- Compose functions numerically, algebraically, and graphically.
- Find $f(x)$ and $g(x)$ given $h(x) = f(g(x))$ by recognizing the inside and outside function.
- Solve equations explicitly for a given variable.
- Perform transformations numerically and graphically given a parent graph.
- Model real-life situations using functions (including particle of motion problems given position vs time graph).

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, numerical, Algebraic*. Pearson.

Suggested Resources: Desmos, Geogebra, Albert.io

UNIT 2: Polynomial, Power & Rational Functions

Summary and Rationale

In this unit, students will review polynomial functions, power functions and rational functions. Students will continue to use transformations to sketch graphs and analyze graphs using end behavior, domain, range, symmetry, intervals of increasing/decreasing and intervals where $f(x) > 0$ and $f(x) < 0$. Additionally, students will identify the relationship between the end of behavior of a function and its horizontal asymptote, a preview to evaluating infinite limits.

Recommended Pacing

10 days

State Standards

Standard F-IF Interpreting Functions

CPI #	Cumulative Progress Indicator (CPI)
6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
7a	Graph linear and quadratic functions and show intercepts, maxima, and minima
7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Standard N-CN The Complex Number System

CPI #	Cumulative Progress Indicator (CPI)
	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Standard A-APR Arithmetic with Polynomials and Rational Expressions

CPI #	Cumulative Progress Indicator (CPI)
6	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system
7	(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Instructional Focus

Unit Enduring Understandings

- Order matters when composing functions.
- Basic rules of equality are the key to ensure accuracy in mathematics.
- Rational functions have asymptotes
- Characteristics allow you to make predictions.
- Relations and Functions can be represented graphically, numerically, analytically, or verbally.

Unit Essential Questions

- Besides checking our work, what are some strategies to help us gain confidence that we are solving a problem correctly?
- What does y approach as x approaches infinity or negative infinity?
- What attributes are necessary when visualizing a function?
- When is it more appropriate to analyze a function algebraically? Graphically?

Objectives

Students will know:

- Only linear functions have a constant rate of change.
- Find the average rate of change of a linear function.
- Basic shapes of linear, absolute value, quadratic functions, and polynomial functions.
- A power function is a function that can be represented in the form, $f(x) = ax^n$ where *a* and *n* are real numbers.
- Rational exponents can be written in radical notation.
- The general shape of the graphs $y = \sqrt[m]{x}$, where *m* is an even number and $y = \sqrt[n]{x}$, where *n* is an odd number.
- The multiplicity of roots in relation to the behavior of a polynomial graph (changing signs versus not changing signs).
- The domain must be found before rewriting/simplifying a function.
- When and where a rational function produces a point of discontinuity and when and where it produces a vertical asymptote.
- A complex number is the sum of a real part and imaginary part. *
- Complex conjugate solutions always come in pairs.*
- The fundamental theorem of algebra.
- Relative growths of functions. (For example, $y = x^4$ grows faster than $y = x^2$).
- When the graph of a rational function has a horizontal asymptote, $y = b$, where b is a constant, the output values of the rational function get arbitrarily close to b and stay arbitrarily close to b as input values approach $\pm\ \infty$. (Refrain from using limit notation).
- Near a vertical asymptote, $x = a$, of a rational function, the values of the polynomial in the denominator are arbitrarily close to zero, so the values of the rational function increase or decrease without bound.
- **Vocabulary**: Power function, multiplicity, end behavior, limits, asymptotes, Rule of Four.

Students will be able to:

- Find the average rate of change of a linear function.
- Apply all types of factoring methods to polynomials.
- Define *i* and complete operations with $a + bi.$ *
- When suitable factorizations are available or with technology, identify key characteristics of a polynomial function related to zeros using the Fundamental Theorem of Algebra.
- Find the end behavior of a function analytically.
- Graph polynomial functions using end behavior, symmetry (if any), and multiplicity of roots.
- Solve radical equations algebraically.
- Compare $f(x)$ to $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for power/root functions and rational functions.
- Given an equation, find the horizontal asymptote using the end behavior of the function.
- Find the domain of rational functions analytically.
- Graph rational functions using a slant asymptote.
- Create a function given essential information.
- Rewrite rational expressions in equivalent forms. (Furthermore, write expressions as a sum/difference of power functions.

- Add and subtract rational expressions by finding LCD.
- Solve rational equations algebraically.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, numerical, Algebraic*. Pearson.

Suggested Resources: Desmos,Geogebra,
Albert.io

UNIT 3: Logs & Exponential Functions with Applications

Summary and Rationale

This unit explores three interrelated families of functions: exponential, logistic, and logarithmic functions. The unit will begin with the discussion of inverses as they specifically examine exponential and logarithmic functions, among some others previous functions. Within the unit, students will begin to examine transcendental functions, which go beyond the algebraic functions as seen in previous coursework. Students will begin their work of these analytic functions as seen in a wide variety of applications such as modeling growth and decay over time both unrestricted and restricted. Modeling and problem solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

Recommended Pacing

10 days

State Standards

Standard F-IF Interpreting Functions

CPI #	Cumulative Progress Indicator (CPI)
7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude
8b	Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t , y = (0.97)t , y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Standard F-BF Building Functions

CPI #	Cumulative Progress Indicator (CPI)
5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

Standard F-LE

CPI #	Cumulative Progress Indicator (CPI)
4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to ab^{ct}= d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
5	Interpret the parameters in a linear or exponential function in terms of a context.

Standard A-REI Reasoning with Equations and Inequalities

CPI #	Cumulative Progress Indicator (CPI)
11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Instructional Focus

Unit Enduring Understandings

- Analyzing trends predict behavior.
- Relationships vary by situation.

- Changing one or many pieces can change the whole.
- Procedures can be reversed.
- Logarithmic functions and exponential functions are inverses of one another.
- Appreciation and depreciation affect the rate of growth.

Unit Essential Questions

- Why are predictions beneficial?
- What is the effect of interchanging parts?
- When is the correct answer not the best solution?
- How do you write and graph exponential and logarithmic functions?
- How do inverse functions manifest themselves when solving equations?

Objectives

Students will know:

- The difference between a power function and exponential function.
- Exponential and logistic functions model many growth patterns, including the growth of human and animal populations
- What an inverse function is and how to find it
- Logarithmic functions are used in many applications, including the measurement of the relative intensity of sounds
- The Richter Scale, pH, and Newton's Law of Cooling are among the most important uses of logarithmic and exponential functions
- The mathematics of finance is key for money management-- including simple interest, compound interest, annual percentage yield, annuities, loans, and mortgage.
- Relative growths of functions. (For example, $y = 2^x$ grows faster than $y = x^{99}$ or $y = x$ grows faster than $y = ln(x)$).
- Exponential and logarithmic functions are inverses of one another, therefore, one produces a horizontal asymptote while the other produces a vertical asymptote, one has an *x*-intercept while the other has a *y*-intercept, the domain and range are interchanged.
- The logarithmic function with base 10 is called the common logarithmic function.
- Exponential functions model many types of unrestricted growth; logistic functions model restricted growth, including the spread of disease and the spread of rumors
- Euler's number e is a very important irrational number used in mathematics approximated by 2.718
- **Vocabulary:** domain/range, even/odd/neither, increasing/decreasing, end behavior, common logarithm, natural logarithm, Euler's number, annual percentage yield (APY), present value, future value, annuity, annual percentage rate(APR)

Students will be able to:

- Find an inverse function both algebraically and graphically with restrictions if necessary.
- Verify that functions are inverses
- Use the left and right end behavior to identify the horizontal asymptote of exponential functions.
- Graph exponential and logarithmic functions
    - Recognize, evaluate, and graph exponential functions with base e.
    - Recognize, evaluate, and graph natural logarithmic functions.
    - Analyze domain, range, increasing/decreasing, asymptotes, intercepts and end behavior
- Compare $f(x)$ to $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for exponential and logarithmic functions
- Make graphical connections from the equation.
- Find the domain of a logarithmic function algebraically.

- Recognize the difference between the graphs of $f(x) = ln(5)$ and $g(x) = ln(x)$
- Identify the type of function: expo growth, expo decay, log, ln, logistic, etc.
- Use exponential modeling for growth (doubling), decay (half-life), compound interest
- Use the different properties (basic, inverse, etc) of logs to evaluate, condense, and expand logs
- Use the change of base formula to rewrite and evaluate logarithmic expressions.
- Simplify, expand, and condense logarithmic expressions.
- Apply inverse properties to solve problems involving exponential, logistic and logarithmic equations
    - Solve exponential equations using properties of exponents, 'taking the log' of both sides, or converting to a log equation.
    - Solve logarithmic equations using properties of logarithms or exponentiating both sides.
- Derive exponential growth and decay equations that model real world situations.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, numerical, Algebraic*. Pearson.

Suggested Resources: Desmos,Geogebra,
Albert.io

UNIT 4: Analysis of Transcendental Functions

Summary and Rationale	
This unit will be used to address a variety of advanced algebra skills that have not been explicitly addressed in the student's previous study of algebra. This unit will utilize complex functions that are composed of one or more simple algebraic and/or transcendental functions to sketch their graphs and solve equations and inequalities. This unit is explicitly for the presentation of skills and complexity of problem solving that students will utilize extensively and come to rely on in their first calculus course.	
Recommended Pacing	
8 days	
State Standards	
Standard F-BF Building Functions	
CPI #	**Cumulative Progress Indicator (CPI)**
1c	(+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
4b	(+) Verify by composition that one function is the inverse of another.
Standard A-CED Creating Equations	
CPI #	**Cumulative Progress Indicator (CPI)**
3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
Standard A-REI Reasoning with Equations and Inequalities	
CPI #	**Cumulative Progress Indicator (CPI)**
7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = –3x and the circle* $x^2 + y^2 = 3$.
10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
Instructional Focus	
Unit Enduring Understandings	
• Order matters when composing functions. • Patterns emerge when identifying the implied domains of composite functions. • Solving nonlinear inequalities requires finding all locations where f(x) changes sign. • A rough sketch of a function can be very useful in problem-solving. • Graphing a composite function can be achieved with intermediate steps. • The student will develop confidence and tenacity when approaching lengthy and intricate math problems. By breaking down a problem into its component parts, analyzing and resolving each part, and then reassembling the whole, the student will develop a sense that no problem is beyond his or her grasp. • The student will begin to understand that the calculator is a tool to supplement and clarify mathematical thinking, not to replace it, and that answers from the calculator need to be anticipated and interpreted	
Unit Essential Questions	

- What conditions need to be considered when identifying the implied domain?
- What attributes are necessary when visualizing a function?
- How does the absolute value affect the graph of a function?
- When is it best to solve a problem algebraically or graphically?
- Why are only a subset of values needed when solving a nonlinear inequality?

Objectives

Students will know:

- The implied domain of a function is the largest possible subset of real numbers where each member of the subset yields a real number when the function is applied to it.
- The implied domain of a function is not all real numbers if:
  - Dividing by zero
  - Taking an even root of a negative number
  - Evaluating the logarithm of a value ≤ 0
- A function can possibly change sign (from + to - or vice versa) when f(x) = 0 or when f(x) is undefined.
- Evaluating a non-linear inequality first requires the evaluation of values where f(x) = 0 or is undefined.
- It is necessary to identify the domain of a composition of two functions before graphing the composition.
- The effect of an absolute value transformation to the graph of a function, y = |f(x)| and y = f(|x|).

Students will be able to:

- Find the implied domain of a function represented algebraically.
- Solve equations using a variety of advanced algebra skills.
- Create sign charts to determine where a function changes signs.
- Solve nonlinear inequalities algebraically and graphically.
- Identify the domains of composite functions.
- Find the end behavior of functions constructed using a combination of algebraic and transcendental functions. (For example, $f(x) = \frac{e^x}{x^2}$).
- Graph compositions of functions constructed using a combination of algebraic and transcendental functions by applying the following strategies:
  - Recognize the parent graph and any transformations, if possible
  - Find domain and range
  - Find intercepts
  - Find end behavior
  - Determine any symmetry
- Graph absolute value transformations of composite functions, y = |f(x)| and y = f(|x|).
- Identify attributes of composite functions (i.e. domain, range, when > or < 0, increasing/decreasing, maximums/minimums) using their algebraic and graphical representations.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, numerical, Algebraic*. Pearson.

Suggested Resources

Circuit Training: Domain and Range (Precalc Level) by Virge Cornelius

Unit 5: Geometric Trigonometry

Summary and Rationale	
This unit will build from student's Algebra 2 knowledge of trigonometry including use of degrees and radians, evaluating trigonometric functions at any angle, and right triangle trig relationships. In this unit, right triangle trigonometry is used with the six trigonometric functions to solve triangles. This unit will extend students' knowledge of trigonometry to include CHOSHACAO and the Law of Sines/Cosines to solve beyond right triangles.	
Recommended Pacing	
9 days	
State Standards	
Standard G-SRT Similarity, Right Triangles, and Trigonometry	
CPI #	**Cumulative Progress Indicator (CPI)**
6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7	Explain and use the relationship between the sine and cosine of complementary angles.
8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
9	(+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10	(+) Prove the Laws of Sines and Cosines and use them to solve problems
11	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Standard F-TF Trigonometric Functions	
CPI #	**Cumulative Progress Indicator (CPI)**
2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3	(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for πx, π+x, and 2π–x in terms of their values for x, where x is any real number.
4	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
6	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★
Instructional Focus	
Unit Enduring Understandings	
• Patterns and relationships can be represented graphically, algebraically, symbolically, or verbally. • Trigonometric functions can be used to describe periodic or cyclic phenomena.	
Unit Essential Questions	
• What connections exist between geometry and trigonometry? • Can we extend trigonometric relationships beyond the right triangle? • What are some of the important properties of trigonometric functions?	

- How are geometric relationships used to analyze periodic behavior?

Objectives

Students will know:

- SOHCAHTOA, CHOSHACAO and how to use to solve for lengths and angles
- The Law of Sines can be used to compute the remaining sides of a triangle when two angles and a side are known.
- The Law of Cosines relates the lengths of the sides of a triangle to the cosine of one of its angles.
- A variety of real-world problems can be solved using values of trigonometric functions including bearings.

Students will be able to:

- Convert decimal degrees to DMS (degrees, minutes, second) and vice versa
- Evaluate the six trigonometric functions for special angles using a right triangle.
- Evaluate inverse functions using a right triangle.
- Solve right triangles using their knowledge of special right triangles, Pythagorean Theorem and trigonometric ratios.
- Use the Law of Sines to solve oblique triangles (AAS and ASA).
- Use the Law of Sines to solve oblique triangles (SSA).
- Use the Law of Cosines to solve oblique triangles (SSS or SAS).
- Find the area of oblique triangles
- Use trigonometric functions to model and solve real-world problems.
- Solve application problems including angle of elevation and depression, or bearing of an object

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, numerical, Algebraic*. Pearson.

Suggested Resources: Desmos,Geogebra,
Albert.io

Unit 6: Analytic Trigonometry

Summary and Rationale

In this unit the definitions of the six trigonometric functions are extended so that they can apply to any angle. The six trigonometric functions are defined based on their location of an angle in standard position, both in degrees and radians, when applied to a coordinate plane. Students will begin with the definition of trigonometric functions applied to acute angles with the sides of a right triangle and then apply this to a coordinate plane to derive new definitions of trigonometric functions based on their (x, y) point on a cartesian coordinate plane. Extending trigonometric functions beyond triangle ratios opens up a new world of applications such as using trigonometric functions to represent periodic behavior (explored in Unit 7) and on the unit circle to form trigonometric identities (explored in Unit 8).

Recommended Pacing

9 days

State Standards

Standard F-TF Trigonometric Functions

CPI #	Cumulative Progress Indicator (CPI)
1	Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3	Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for πx, π+x, and 2π–x in terms of their values for x, where x is any real number.
4	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions
5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

Standard G Geometry

CPI #	Cumulative Progress Indicator (CPI)
5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Instructional Focus

Unit Enduring Understandings

- Reason quantitatively using all representations of angle measures (One rotation = 2π radians = 360^{o}).
- Make sense of problems involving applications of angle measures.
- Trigonometry is applicable to all angles.
- Look for and make use of the structure of the unit circle (Examples: All coterminal angles are related by a whole number of rotations, the Reference Angle Theorem).

Unit Essential Questions

- What is the difference between degrees and radians?
- What is the difference between linear and angular speed?
- What patterns emerge from the Unit Circle?
- How do the analytic trig definitions emerge from the right triangle trig definitions?
- What are the limitations of right triangle trigonometry?

Objectives

Students will know:

- The definition of an angle in standard position on a cartesian coordinate plane.
- Both degrees and radians are utilized to measure angles.
- One radian is equal to the central angle that intercepts an arc equal in length to the radius of the circle.
- The definition of coterminal angles.
- The definition of a reference angle.
- Angular speed is independent of the radius while linear speed is dependent on the radius
- The arc length formula ($s = \theta r$) can be derived using the circumference of a circle.
- The sign of each trigonometric function is dependent on the quadrant in which the angle terminates.
- Trigonometric functions of special angles can be found using the reference angle theorem.
- The components of a 30-60-90^{o} and 45-45-90^{o} triangle.
- Quadrantal angles have a terminal side along one of the coordinate axes.
- Trigonometric functions of any angle can be evaluated using a point (x,y) on the terminal side.

Students will be able to:

- Draw angles in standard position using degrees and radians.
- Reason with angle measures (i.e. identify that 3 radians is an angle measure that would terminate in quadrant 2 because π radians terminates on the negative x-axis and this is > 3).
- Identify and find coterminal angles.
- Identify and find reference angles.
- Calculate linear and angular speed.
- Calculate arc length and sector area.
- Evaluate all six trigonometric functions given a point or equation of a line.
- Evaluate all six trigonometric functions given the ratio of one trigonometric function.
- Evaluate all six trigonometric functions of special angles and quadrantal angles.
- Evaluate all six trigonometric functions of any angle using a calculator.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, numerical, Algebraic*. Pearson.

Suggested Resources: Desmos,Geogebra,
Albert.io

Unit 7: Graphs of Trigonometric Functions

Summary and Rationale

In this unit students will use their knowledge of the unit circle and coterminal angles to get an appreciation for the periodic nature of trigonometric functions. The structure of these functions will be observed by plotting them on the cartesian coordinate plane where the x-axis is the angle measure and the y-axis is the output of the trigonometric function. Graphing all six trigonometric functions will illustrate the periodic behavior of these functions. New terminology will be introduced to describe the graphs of these trigonometric functions. These functions, in particular sine and cosine, will then be used to model real world applications that exhibit periodic behavior. Students will utilize their knowledge of inverse functions to graph the inverse trigonometric functions and identify their domain restrictions.

Recommended Pacing

10 days

State Standards

Standard F-TF Trigonometric Functions

CPI #	Cumulative Progress Indicator (CPI)
5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
6	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Instructional Focus

Unit Enduring Understandings

- Trigonometric functions are used to model periodic behavior.
- Look for and make use of structure in the graphical and algebraic representation of trigonometric functions.
- Interpret key features of a trigonometric function algebraically and graphically.
- Use the concept of a function to identify the domain restrictions for inverse trigonometric functions.

Unit Essential Questions

- What relationships do you see between the graphs of different trigonometric functions?
- What do the different parts of the trigonometric function tell you about the graph?
- What applications exhibit periodic/cyclical behavior?
- What is the difference between inverse and reciprocal trigonometric functions?

Objectives

Students will know:

- The basic shapes and behaviors for the parent graphs of the six trigonometric functions.
- The vocabulary used to describe key features of a trigonometric function (period, amplitude, midline, phase shift).
- The graph of a trigonometric function can have multiple algebraic representations.
- Real world applications that are modeled by trigonometric functions (Examples: Ebb and Flow of Tides, Harmonic Motion).
- The relationship between trigonometric functions and their inverses.
- The difference between inverse trigonometric functions and reciprocal trigonometric functions.
- Inverse trigonometric functions have only one output for a given input.
- Domain restrictions of inverse trigonometric functions.

Students will be able to:

- Graph all six parent trigonometric functions.
- Connect the new vocabulary used to describe trigonometric functions to the standard transformation form utilized in previous algebra courses. (Example: Amplitude change is equivalent to a vertical stretch/shrink.)
- Graph trigonometric functions using the standard transformation form [f(x) = a*sin(b(x-h)) + k]
- Write multiple trigonometric functions for a single graph.
- Construct trigonometric functions given their attributes.
- Solve trigonometric equations graphically using graphing calculators.
- Graph inverse trigonometric functions and identify their domain restrictions.
- Evaluate inverse trigonometric functions.

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, numerical, Algebraic*. Pearson.

Suggested Resources: Desmos,Geogebra,
Albert.io

Unit 8: Trigonometry Identities and Equations

Summary and Rationale

In this unit, students will learn about different properties and identities that connect the trigonometric functions in a variety of ways. This unit focuses more on theory and proof, exploring where the properties of these functions lead. Students will learn how to use these identities to simplify expressions, prove identities and solve simple and complex trigonometric equations.

Recommended Pacing

10 days

State Standards

Standard F-TF Trigonometric Functions

CPI #	**Cumulative Progress Indicator (CPI)**
8	Prove the Pythagorean identity sin^2 (θ) + cos^2(θ) = 1 and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9	(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Instructional Focus

Unit Enduring Understandings

- The skills used to manipulate algebraic expressions are needed to simplify trigonometric expressions
- The trigonometric identities can be written in multiple equivalent forms

Unit Essential Questions

- How can algebra be used to simplify trigonometric expressions?
- What is the relationship between the pythagorean theorem and the fundamental identities?
- What is the relationship between the trigonometric identities and the ratios they represent?

Objectives

Students will know:

- Fundamental trigonometric identities (pythagorean/cofunction/odd-even) and how to use them
- Sum/Difference Formulas and how to use them
- Double-Angle & Half-Angle Formulas and how to use them
- How multiple angles affect solutions to a trigonometric equation
- The difference between finding all solutions to a trigonometric equations and finding solutions on a closed interval

Students will be able to:

- Use trigonometric formulas and identities to evaluate trigonometric expressions
- Use trigonometric formulas and identities to simplify trigonometric expressions
- Use trigonometric formulas to prove trigonometric identities
- Use trigonometric formulas to solve trigonometric equations
- Solve basic trigonometric equations in radians and degrees
- Solve trigonometric equations involving multiple angles with an emphasis on radians
- Solve trigonometric equations on a closed interval vs an open interval

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, numerical, Algebraic*. Pearson.

Suggested Resources: Desmos,Geogebra, Albert.io

Unit 9: Analytic Geometry*

Summary and Rationale

In this unit students will learn about applications for trigonometry. Students will learn how trigonometry can relate to parametric equations. Students will be introduced to the polar coordinate system and explore what rectangular graphs look like when translated into polar coordinates. In this unit, students will also learn about conic sections which is an extension of linear equations to quadratic equations. Students will explore graphs of parabolas, circles, ellipses and hyperbolas.

Recommended Pacing

10 days

State Standards

Standard N-CN The Complex Number System

CPI #	Cumulative Progress Indicator (CPI)
4	(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

Standard G-GPE Expressing Geometric Properties with Equations

CPI #	Cumulative Progress Indicator (CPI)
1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2	Derive the equation of a parabola given a focus and directrix.
3	(+)Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Instructional Focus

Unit Enduring Understandings

- Analytic geometry is a systematic approach that can be used to solve algebra problems geometrically and geometry problems algebraically

Unit Essential Questions

- What is the benefit of rectangular and polar coordinates?
- What applications do conic sections have for mathematics?

Objectives

Students will know:

- Definition of parametric equations
- What a polar coordinate looks like and how to graph them
- Different types of polar curves (rose, limacon, spiral, lemniscate etc)
- Definition of a conic section
- How foci, and directrix relate to the graph of a parabola, ellipse or hyperbola
- Applications for conic sections

Students will be able to:

- Sketch the graph of a parametric curve
- Eliminate the parameters of parametric equations
- Apply parametric equations to real world situations
- Graph a polar coordinate and find equivalent polar coordinates

- Convert between polar and rectangular form of a coordinate and an equation
- Test for symmetry in a polar graph
- Sketch the graph of polar curves (rose, limacon, spiral, lemniscate etc)
- Analyze the graphs of polar curves by finding critical values (domain, range, continuity, symmetry etc)
- Classify conic graphs from equations
- Sketch the graph of a parabola from the directrix and focus
- Identify critical values of a parabola, ellipse & hyperbola (foci, vertices, major/minor axis, focal axis etc)
- Sketch the graph of an ellipse or hyperbola given critical values (foci/vertices/covertices/focal axis)
- Write the equation of a conic graph given critical values
- Apply conic equations to real word scenarios

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, numerical, Algebraic*. Pearson.

Suggested Resources: Desmos,Geogebra,
Albert.io

Unit 10: Sequences and Series

Summary and Rationale

In this unit, students will extend their knowledge of sequences to the study of series. Students were introduced to arithmetic and geometric sequences in Algebra 1. In this unit, students will calculate the nth term and nth partial sum of arithmetic and geometric sequences for both real world and mathematical situations. Students will represent series using sigma notation.

Recommended Pacing

4 Days (Time permitting)

State Standards

Standard F-BF Building Functions

CPI #	**Cumulative Progress Indicator (CPI)**
2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Standard F-LE Linear and Exponential Models

CPI #	**Cumulative Progress Indicator (CPI)**
2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standard A-SSE Seeing Structures in Expressions

CPI #	**Cumulative Progress Indicator (CPI)**
4	Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments*.★

Instructional Focus

Unit Enduring Understandings

- Some sequences can be modeled with a function rule that you can use to find any term of the sequence.
- Sequences and series are discrete functions whose domain is the set of whole numbers.

Unit Essential Questions

- How can patterns be represented?

Objectives

Students will know:

- Since sequences are not continuous, but rather discrete, they do not represent functions.
- Arithmetic and Geometric Sequences
- Geometric Sequences resemble an exponential function. (Difference is that the common ratio in a sequence may be less than zero).
- Infinite sequence is called a series.
- If the partial sums converge to a number then the infinite series converges to that value.

Students will be able to:

- Identify the rule for the n^{th} term given a sequence.
- Write out the first few terms of an infinite series (given the sigma notation).

- Determine the sum of a finite series (given the sigma notation).
- Identify the ratio of a geometric series.
- Determine if a geometric series converges or diverges.
- Find partial sums of an infinite series (For example, the first 5 partial sums).
- Use the partial sums to decide if a series converges or diverges
- Use graphing technology to see the graph of a sequence and series (Desmos recommended)

Resources

Core Text: Demana, F. D., Waits, B. K., Foley, G. D., Kennedy, D., & Bock, D. E. (2019). *Precalculus: Graphical, numerical, Algebraic*. Pearson.

Suggested Resources: Desmos,Geogebra,
Albert.io