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AP Calculus BC

Content Area:	**Mathematics**
Grade Span:	**11-12**
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COURSE OVERVIEW

Description

AP Calculus BC is a rigorous course designed for students with outstanding skills and interests in mathematics who want to gain college credit by taking the College Board Advanced Placement examination in May. Both practical and theoretical approaches are presented at an accelerated pace and explorations are used extensively to engage and motivate students. Topics presented in this course include functions, limits, derivatives, integrals, and numerical approximations. Concepts are introduced using the Rule of Four (graphically, numerically, analytically, and verbally) so that students can continually demonstrate the connections between these representations. Emphasis is placed on understanding and using the mathematical modeling process to set up and solve a variety of problems.

Graphing calculators are used throughout the course and are required on some portions of the exam. This tool is used to develop conjectures, connect concepts to their visual representation, solve problems and critically interpret and accurately report information.

Goals

In addition to the content standards, skills, and concepts set forth by College Board, this course will also focus on the AP Calculus AB and BC Mathematical Practices.The AP Calculus AB and BC Mathematical Practices are outlined below:

1. Implementing mathematical processes: Determine expressions and values using mathematical procedures and rules
2. Connecting Representations: Translate mathematical information from a single representation or across multiple representations.
3. Justification: Justify reasoning and solutions.
4. Communication and Notation: Use correct notation, language, and mathematical conventions to communicate results or solutions.

Scope and Sequence

Unit	Topic	Length (Blocks)
Unit 0	Prerequisites for Calculus	3
Unit 1	Limits and Continuity	12
Unit 2	Differentiation: Definition and Basic Derivative Rules	14
Unit 3	Advanced Differentiation Techniques	10
Unit 4	Applications of Differentiation	17
Unit 5	Integration and Accumulation of Change	20
Unit 6	Application of integration	11
Unit 7	Differential Equations	11
Unit 8	Infinite Sequences and Series	18
Unit 9	Parametric Equations and Polar Coordinates and Vector Valued- Functions	10

Resources

Core Text: *Calculus-Graphical, Numerical, Algebraic* (2012). Finney, Demana, et al. Prentice Hall

Suggested Resources

UNIT 0: Prerequisite for Calculus

Summary and Rationale

Functions and graphs form the basis for understanding mathematics and applications. This unit introduces all the elementary functions to be used in the course. Although the functions are probably familiar, the graphical, numerical, verbal, and analytical (Rule of Four) approach to their analysis may be new.

Recommended Pacing

3 days

Instructional Focus

Unit Enduring Understandings

- Algebraic representation can be used to generalize patterns and relationships
- Patterns and relationships can be represented graphically, numerically, symbolically, or verbally
- Mathematical models can be used to describe and quantify physical relationships
- Equations that model real-world data allow you to make predictions about the future.

Unit Essential Questions

- How can change be best represented mathematically?
- How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?
- How are patterns of change related to the behavior of functions?
- How can we use mathematical models to describe physical relationships?
- What type of equation would model specific real-world data?

Objectives

Students will know:

- Point-Slope form and its derivation
- Definition of even function and odd function
- Definition of an inverse of a function
- Properties of logarithms
- Parent graphs of common functions, such as, quadratic, rational, exponential, logarithmic, and trigonometric
- The difference between solving a linear inequality and nonlinear inequality

Students will be able to:

- Write the equation of a line using Point-Slope Form
- Identify the domain and range of a function using its graph or equation using interval notation
- Recognize even and odd functions using its graph or equation
- Write and evaluate composition of two functions
- Solve exponential growth and exponential decay problems.
- Find the inverse of a function graphically and algebraically
- Apply properties of logarithms
- Evaluate trigonometric functions
- Graph functions using a variety of strategies, such as, transformations, symmetry, end behavior, asymptotes.
- Solve nonlinear inequalities graphically and algebraically
- Solve system of equations
- Solve for a given variable
- Use a graphing calculator to solve an equation and graph functions in a specified window to find zeros, points of intersection and solve nonlinear inequalities

Resources

Core Text: *Calculus-Graphical, Numerical, Algebraic* (2012). Finney, Demana, et al. Prentice Hall

Suggested Resources: *Calculus* (2009) Hughes-Hallet, Gleason, et al, John Wiley & Sons, Inc.; khanacademy.org; desmos.com; collegeboard.org; graphing calculators

UNIT 1: Limits and Continuity

Summary and Rationale
The limit is a fundamental concept in higher math. A theoretical understanding of the limit allows us to work with infinitesimally small values, building the bridge from estimated slopes and areas to the exact values found by applying derivatives and integrals. Students must have a solid, intuitive understanding of limits and be able to compute various limits, including one-sided, limits at infinity, and infinite limits. They will apply the Rule of Four by working with tables and graphs in order to estimate the limit of a function at a point. Students must also understand how limits are used to determine continuity, a fundamental property of a function. A graphing calculator is utilized to find limits numerically using the table feature and graphically using the Trace feature. A calculator will also help students determine the asymptotic behavior of a function and investigate the continuity of a function.

Recommended Pacing
12 days

AP Big Ideas	
Standard CHA - Change	
#	**Outcomes**
1.A	Interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant.
Standard LIM - Limits	
#	**Outcomes**
1.A	Represent limits analytically using correct notation.
1.B	Interpret limits expressed in analytic notation
1.C	Estimate limits of functions.
1.D	Determine the limits of functions using limit theorems.
1.E	Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.
2.A	Justify conclusions about continuity at a point using the definition.
2.B	Determine intervals over which a function is continuous.
2.C	Determine values of x or solve for parameters that make discontinuous functions continuous, if possible.
2.D	Interpret the behavior of functions using limits involving infinity
Standard FUN - Functions	
#	**Outcomes**
1.A	Explain the behavior of a function on an interval using the Intermediate Value Theorem.

Instructional Focus

Unit Enduring Understandings

- Reasoning with definitions, theorems, and properties can be used to justify claims about limits.
- Identify mathematical information from graphical, symbolic, numerical and/or verbal representations.
- Apply appropriate mathematical procedures with and without technology.
- Identify an appropriate mathematical rule or procedure to evaluate a limit based on the classification of a given expression.
- Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied (Proving Continuity, Squeeze Theorem)

- Provide reasons or rationales for solutions or conclusions (Intermediate Value Theorem)

Unit Essential Questions

- How does knowing the value of a limit, or that a limit does not exist, help you to make sense of interesting features of functions and their graphs?
- How does the value of a function at a point affect the limit of the function at that point?
- How is continuity at a point related to the limit of the function at that point?
- When there are multiple approaches, how should you choose the best method?

Objectives

Students will know:

- The informal definition of a limit. Given a function, f, the limit of f(x) as x approaches c is a real number R if f(x) can be made arbitrarily close to R by taking x sufficiently close to (but not equal to) c.
- Limit notation.
- The concept of a limit includes one sided limits.
- Limits can be evaluated from the graph of a function.
- Cases where a limit does not exist:
    - If a function is unbounded.
    - If a function is oscillating.
    - If the one sided limits are not equal.
- Numerical information can be used to estimate limits.
- Algebraic properties of limits.
- Evaluate limits using algebraic manipulation.
- The limit of a function may be found by using the squeeze theorem.
- Definition of continuity at a point. A function f is continuous at x = c provided that f(c) exists, the limit of f(x) as x approaches c exists, and f(c) equals the limit of f(x) as x approaches c.
- Types of discontinuities:
    - Removable
    - Jump
    - Infinite (due to vertical asymptote)
- A function is continuous on an interval if the function is continuous at each point on the interval.
- The definition of a continuous function.
- If a limit of a function exists at a discontinuity then it is possible to remove the discontinuity by defining or redefining a value at that point.
- Asymptotic and unbounded behavior of functions can be described and explained using limits.
- Limits at infinity describe end behavior.
- Relative magnitudes of functions and their rates of change can be compared using limits.
- The Intermediate Value Theorem.

Students will be able to:

- Represent limits analytically using correct notation.
- Estimate limits of functions from a graph.
- Estimate limits of functions from a table.
- Determine the limits of functions using algebraic properties of limits.
- Evaluate limits algebraically using equivalent expressions for the function when needed.
- Select the appropriate procedure for evaluating limits.
- Determine the limits of functions using the squeeze theorem.
- Justify conclusions about continuity at a point using the definition.
- Determine intervals over which a function is continuous.
- Determine values of x or solve for parameters that make discontinuous functions continuous, if possible.
- Interpret the behavior of functions using limits involving infinity.

- Explain the behavior of a function on an interval using the Intermediate Value Theorem.
- Use a graphing calculator to solve problems, experiment, interpret results, and support conclusions.

Resources

Core Text: *Calculus-Graphical, Numerical, Algebraic* (2012). Finney, Demana, et al. Prentice Hall

Suggested Resources: *Calculus* (2009) Hughes-Hallet, Gleason, et al, John Wiley & Sons, Inc.; khanacademy.org; desmos.com; collegeboard.org; graphing calculators

UNIT 2: Differentiation: Definition and Basic Rules

Summary and Rationale

Using derivatives to describe the rate of change of one variable with respect to another variable allows students to understand change in a variety of contexts. With their understanding of functions, students will recognize that the slopes of the tangents at the given points represent a relationship between the two quantities. Students will call this function the derivative of a function. The derivative is the key to modeling instantaneous change. Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. Applications of the derivative include finding the slope of a tangent line to a graph at a point, analyzing the graph of a function, and solving problems involving rectilinear motion.

Recommended Pacing

14 days

AP Big Ideas

Standard CHA - Change

#	Outcomes
2.A	Determine average rates of change using difference quotients.
2.B	Represent the derivative of a function as the limit of a difference quotient.
2.C	Determine the equation of a line tangent to a curve at a given point.

Standard FUN - Functions

#	Outcomes
2.A	Explain the relationship between differentiability and continuity.
3.A	Calculate derivatives of familiar functions.
3.B	Calculate derivatives of products and quotients of differentiable functions.

Standard LIM - Limits

#	Outcomes
3.A	Interpret a limit as a definition of a derivative.

Instructional Focus

Unit Enduring Understandings

- Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals.
- Recognizing that a function's derivative may also be a function allows us to develop knowledge about the related behaviors of both.
- Recognizing opportunities to apply derivative rules can simplify differentiation.
- Relationships can be represented graphically, numerically, analytically, or verbally (rule of four).

Unit Essential Questions

- Can change occur at an instant?
- What is a derivative and how does it differ in various situations?
- What can you predict about f given f'?
- When there are multiple approaches, how should you choose the best method?
- What are the advantages of having different ways to represent a derivative?

Objectives

Students will know:

- Calculus uses limits to understand and model dynamic change.
- The difference quotient expresses the average rate of change of a function over an interval.
- The instantaneous rate of change of a function at x = a can be expressed by the limit of the difference quotient as x approaches a.
- Notation used to express the derivative (dy/dx, f'(x), y').
- The derivative of a function at a point is the slope of the line tangent to a graph of the function at that point.
- The derivative at a point can be estimated from the information given in tables or graphs.
- The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
- The derivative can be used to solve rectilinear motion problems involving position, speed, velocity and acceleration.
- If a function is differentiable at a point, then it is continuous at that point.
- A continuous function may fail to be differentiable at a point in its domain.
- Power functions are those that can be written as y = ax^n where n is a real number.
- Sums, differences, and constant multiples of functions can be differentiated using derivative rules.
- The specific rules used to find the derivatives for sine, cosine, exponential and logarithmic functions.
- Derivatives of products or quotients of differentiable functions can be found using the product or quotient rule.
- Derivatives of the remaining trigonometric functions (tangent, cotangent, secant, cosecant).

Students will be able to:

- Determine average rates of change using difference quotients.
- Interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant.
- Represent the derivative of a function as the limit of a difference quotient.
- Determine the equation of a line tangent to a curve at a given point.
- Estimate derivatives using tables and graphs.
- Interpret the meaning of a derivative in context.
- Calculate rates of change in applied contexts.
- Explain the relationship between differentiability and continuity.
- Identify power functions.
- Utilize the power rule to evaluate the derivative of a power function.
- Utilize the constant, sum, difference, and constant multiple derivative rules.
- Estimate the derivatives of sine, cosine, exponential, and logarithmic functions to formulate their derivative rules.
- Calculate derivatives of products and quotients of differentiable functions.
- Apply the quotient rules with trig identities to evaluate the derivatives of tangent, cotangent, second and cosecant.
- Use a graphing calculator to solve problems, experiment, interpret results, and support conclusions.

Resources

Core Text: *Calculus-Graphical, Numerical, Algebraic* (2012). Finney, Demana, et al. Prentice Hall

Suggested Resources: *Calculus* (2009) Hughes-Hallet, Gleason, et al, John Wiley & Sons, Inc.; khanacademy.org; desmos.com; collegeboard.org; graphing calculators

UNIT 3: Advanced Differentiation Techniques

Summary and Rationale

In this unit students will learn advanced techniques and further applications for differentiation. Students will add the chain rule to their repertoire of derivative rules in order to differentiate composite functions. The chain rule allows for students to implicitly differentiate functions and solve problems involving related rates. In addition students will explore the relationship between derivatives of inverse functions, higher order derivatives (along with their contextual applications) and L'Hospital's rule for evaluating limits in the indeterminate form.

Recommended Pacing

10 days

AP Big Ideas

Standard FUN - Functions

#	**Outcomes**
3.C	Calculate derivatives of compositions of differentiable functions.
3.D	Calculate derivatives of implicitly defined functions.
3.E	Calculate derivatives of inverse and inverse trigonometric functions.
3.F	Determine higher order derivatives of a function.

Instructional Focus

Unit Enduring Understandings

- The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.
- There are many ways of evaluating the derivative.
- Relationships can be represented graphically, numerically, analytically, or verbally (rule of four).

Unit Essential Questions

- When there are multiple approaches, how should you choose the best method?
- What are the advantages of having different ways to represent a derivative?
- How are the derivatives of inverse functions related?
- In what circumstances is it necessary to apply L'Hospital's rule?

Objectives

Students will know:

- The chain rule provides a way to differentiate composite functions.
- The chain rule is the basis for implicit differentiation.
- The chain rule is the basis for differentiating variables in a related rates problem with respect to the same independent variable.
- The derivative can be used to solve related rates problems; that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.
- Corresponding points of inverse functions have reciprocal slopes (i.e. $f'(a) = 1/g'(b)$ provided that f and g are inverses and $f(a) = b$).
- Right triangle trigonometry can be used to formulate the derivative rules for inverse trigonometric functions.
- Differentiating f' produces f'', provided the derivative of f' exists. This process can be repeated to produce higher order derivatives.
- Limits of indeterminate forms ($0/0$ or ∞/∞) may be evaluated using L'Hospital's Rule.

Students will be able to:

- Calculate derivatives of compositions of differentiable functions.
- Calculate derivatives of implicitly defined functions.
- Calculate related rates in applied contexts.

- Interpret related rates in applied contexts.
- Calculate derivatives of inverse functions.
- Calculate derivatives of inverse trigonometric functions.
- Select the appropriate procedure for calculating derivatives.
- Determine higher order derivatives of a function.
- Determine limits of functions that result in indeterminate forms.
- Use a graphing calculator to solve problems, experiment, interpret results, and support conclusions.

Resources

Core Text: *Calculus-Graphical, Numerical, Algebraic* (2012). Finney, Demana, et al. Prentice Hall

Suggested Resources: *Calculus* (2009) Hughes-Hallet, Gleason, et al, John Wiley & Sons, Inc.; khanacademy.org; desmos.com; collegeboard.org; graphing calculators

UNIT 4: Applications of Differentiation

Summary and Rationale

Differential calculus is a powerful problem-solving tool precisely because of its usefulness for analyzing functions. In this unit students will sketch the graph of f by determining where f is increasing or decreasing and finding inflection points and extreme values. In addition, students should understand and be able to apply the Extreme Value Theorem and the Mean Value Theorem, and be familiar with a variety of real-world applications, including related rates, optimization, and growth and decay models. Finding maximum and minimum values of functions, called optimization, is an important issue in real-world problems.

Recommended Pacing

17 days

AP Big Ideas

Standard CHA - Changes

#	**Outcomes**
3.A	Interpret the meaning of a derivative in context.
3.B	Calculate rates of change in applied contexts.
3.C	Interpret rates of change in applied contexts.
3.D	Calculate related rates in applied contexts.
3.E	Interpret related rates in applied contexts.
3.F	Approximate a value on a curve using the equation of a tangent line.

Standard LIM - Limits

CPI #	**Cumulative Progress Indicator (CPI)**
#	**Outcomes**
4.A	Determine limits of functions that result in indeterminate forms.

Instructional Focus

Unit Enduring Understandings

- Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
- Calculus is a collection of powerful ideas; not a set of rules, formulas and procedures. To learn calculus requires focus on the understanding of a few big ideas, not merely memorizing techniques.
- A function's derivative, which is itself a function, can be used to understand the behavior of the function.
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole
- Mathematical models can be used to describe and quantify physical relationships
- Physical models can be used to clarify mathematical relationships

Unit Essential Questions

- How can we use mathematical models to describe physical relationships?
- How can we use physical models to clarify mathematical relationships?
- How can we use derivatives to solve problems?
- What can you predict about f and f'' given f'?
- How do we measure something that is changing with respect to time?

Objectives

Students will know:

- The first derivative of a function can provide information about the function and its graph including:
    - Intervals where the function is increasing (f' > 0) and decreasing (f' < 0).
    - Location of relative (local) extrema of the function.

- The second derivative of a function provides information about the function and its graph including:
    - Intervals of upward and downward concavity.
    - Points of inflection.
- The graph of a function is concave up (down) on an open interval if the function's derivative is increasing (decreasing) on that interval.
- The second derivative of a function may determine whether a critical point is the location of a relative (local) maximum or minimum.
- When a continuous function has only one critical point on an interval and the critical point corresponds to a relative extrema, then that critical point is also the absolute extrema of the function on the interval.
- The Mean Value Theorem.
- The Extreme Value Theorem.
- Absolute (global) extrema of a function on a closed interval can only occur at critical points or at end points.
- Graphical, numerical and analytical information from f' and f'' can be used to predict and explain the behavior of f.
- The derivative can be used to solve optimization problems.
- The tangent line is the graph of a locally linear approximation of the function near the point of tangency.
- For a tangent line approximation, the function's behavior near the point of tangency may determine whether a tangent line value is an underestimate or overestimate of the corresponding function value.

Students will be able to:

- Justify conclusions about the behavior of a function based on the behavior of its derivatives.
- Determine local extreme values of a function using the derivative the First Derivative Test.
- Sketch functions based on information from f' and f''.
- Justify conclusions about functions by applying the Extreme Value Theorem and the Candidates Test.
- Justify conclusions about functions by applying the Mean Value Theorem.
- Calculate minimum and maximum values in applied contexts or analysis of functions.
- Interpret minimum and maximum values calculated in applied contexts.
- Approximate a value on a curve using the equation of a tangent line.
- Use a graphing calculator to find the derivative, global and local extreme values, inflection points, and intervals where the function is increasing or decreasing.
- Use a graphing calculator to solve problems, experiment, interpret results, and support conclusions.

Resources

Core Text: *Calculus-Graphical, Numerical, Algebraic* (2012). Finney, Demana, et al. Prentice Hall

Suggested Resources

UNIT 5: Integration and Accumulation of Change

Summary and Rationale

This unit establishes the relationship between differentiation and integration using the Fundamental Theorem of Calculus. Students begin by exploring the contextual meaning of areas of certain regions bounded by rate functions. Integration determines accumulation of change over an interval, just as differentiation determines instantaneous rate of change at a point. Students should understand that integration is a limiting case of a sum of products (areas) in the same way that differentiation is a limiting case of a quotient of differences (slopes). Future units will apply the idea of accumulation of change to a variety of realistic and geometric applications.

Recommended Pacing

20 days

AP Big Ideas

Standard CHA - Changes

#	Outcomes
4.A	Interpret the meaning of areas associated with the graph of a rate of change in context.

Standard LIM- Limits

#	Outcomes
5.A	Approximate a definite integral using geometric and numerical methods.
5.B	Interpret the limiting case of the Riemann sum as a definite integral
5.C	Represent the limiting case of the Riemann sum as a definite integral.

Standard FUN- Functions

#	Outcomes
5.A	Represent accumulation functions using definite integrals.
6.A	Calculate a definite integral using areas and properties of definite integrals.
6.B	Evaluate definite integrals analytically using the Fundamental Theorem of Calculus.
6.C	Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives.
6.D	For integrands requiring substitution or rearrangements into equivalent forms: (a) Determine indefinite integrals. (b) Evaluate definite integrals

Instructional Focus

Unit Enduring Understandings

- Definite integrals allow us to solve problems involving the accumulation of change over an interval.
- Definite integrals can be approximated using geometric and numerical methods.
- The Fundamental Theorem of Calculus connects differentiation and integration.
- Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration
- The use of limits allows us to show that the areas of unbounded regions may be finite. *

Unit Essential Questions

- How is integrating to find areas related to differentiating to find slopes?
- How can we use mathematical models to describe physical relationships?
- How can we use physical models to clarify mathematical relationships?
- What does a definite integral represent?
- When there are multiple approaches, how should you choose the best method?

Objectives

Students will know:

- The area of the region between the graph of a rate of change function of the x-axis gives the accumulation of change.
- In some cases, accumulation of change can be evaluated by using geometry.
- If a rate of change is positive (negative) over an interval, then the cumulative change is positive (negative).
- The unit for the area or region defined by rate of change is a unit for the rate of change multiplied by the unit for the independent variable.
- Definite integrals can be approximated for functions that are represented graphically, numerically, analytically, and verbally.
- Depending on the behavior of a function, it may be possible to determine whether an approximation for a definite integral is an underestimate or overestimate for the value of the definite interval.
- The limit of an approximating Riemann sum can be interpreted as a definite internal.
- A Riemann sum, which requires a partition of the interval I, is the sum of the products, each of which is the value of the function at a point in us up in a real multiplied by the length of that's available of the partition
- The definite integral can be used to define new functions
- Fundamental Theorem of Calculus, part 1 and part 2, graphically, numerically, analytically, and verbally.
- Properties of definite integrals.
- An antiderivative of a function f is a function g whose derivative is f.
- Differentiation rules provide the foundation for finding antiderivatives.
- Difference between a definite and indefinite integral.
- For a definite integral, substitution of variables requires corresponding changes to the limits of integration.
- Techniques for finding antiderivatives include rewriting the integrand into an equivalent form.
- An improper integral is an integral that has one or both limits infinite or has an integran that is unbounded in the interval of integration.

Students will be able to:

- Use appropriate units of measure.
- Explain how an approximated value relates to the actual value.
- Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.
- Use appropriate mathematical symbols and notation, e.g., $\int f'(x)\, dx$
- Approximate definite integrals using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
- Determine if an approximation is an underestimate or overestimate.
- Convert a definite integral into the limit of a related Riemann sum and vice versa.
- Extend the definition of a definite integral to functions with removal or jump discontinuities.
- Evaluate integrals algebraically using properties of integrals, reverse power rule, u-substitution, integration by parts, recalling derivative rules, and/or decomposing a rational function into its partial sums. *
- Identify improper integrals.
- Evaluate an improper integral or determine that the integral diverges. *
- Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems.

Resources

Core Text: *Calculus-Graphical, Numerical, Algebraic* (2012). Finney, Demana, et al. Prentice Hall

Suggested Resources:

UNIT 6: Application of integration

Summary and Rationale
In this unit, students will learn how to find the average value of a function, model particle motion and net change, and determine areas, volumes, and lengths* defined by the graphs of functions. Emphasis should be placed on developing an understanding of integration that can be transferred across these and many other applications. Understanding that the area, volume, and length bc only problems studied in this unit are limiting cases of Riemann sums of rectangle areas, prism volumes, or segment lengths* saves students from memorizing a long list of seemingly unrelated formulas and develops meaningful understanding of integration.

Recommended Pacing
11 days

AP Big Ideas	
Standard CHA - Changes	
#	**Outcomes**
4.A	Interpret the meaning of areas associated with the graph of a rate of change in context.
Standard LIM - Limits	
#	**Outcomes**
5.A	Approximate a definite integral using geometric and numerical methods.
5.B	Interpret the limiting case of the Riemann sum as a definite integral.
5.C	Represent the limiting case of the Riemann sum as a definite integral.
6.A	Evaluate an improper integral or determine that the integral diverges.
Standard FUN - Functions	
#	**Outcomes**
5.A	Represent accumulation functions using definite integrals.
6.A	Calculate a definite integral using areas and properties of definite integrals.
6.B	Evaluate definite integrals analytically using the Fundamental Theorem of Calculus.
6.C	Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives.
6.D	For integrands requiring substitution or rearrangements into equivalent forms: (a) Determine indefinite integrals. (b) Evaluate definite integrals.
6.E	For integrands requiring integration by parts: (a) Determine indefinite integrals. (b) Evaluate definite integrals
6.F	For integrands requiring integration by linear partial fractions: (a) Determine indefinite integrals. (b) Evaluate definite integrals.

Instructional Focus
Unit Enduring Understandings
• Definite integrals allow us to solve problems involving the accumulation of change over an interval. • Mathematical models can be used to describe and quantify physical relationships • Physical models can be used to clarify mathematical relationships
Unit Essential Questions

- How can we use mathematical models to describe physical relationships?
- How can we use physical models to clarify mathematical relationships?
- When there are multiple approaches, how should you choose the best method?
- How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?

Objectives

Students will know:

- The formula for the average value of a function over a closed interval.
- A function defined as an integral represents an accumulation of rate of change.
- The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over the interval.
- The definite integral can be used to express information about accumulation and net change in many applied contexts.
- For a particle in rectilinear motion over an interval of time the definite integral of velocity represents a particle displacement over the interval of time and the definite integral of speed represents the particle's total distance traveled over the interval of time.
- Difference between displacement, total distance traveled, and position of a particle at a given time in rectilinear motion.
- Area of regions in the plane or volumes of solids a revolution around the x or y axis can be calculated with definite integrals.
- Area of certain regions in the plane may be calculated using a sum of two or more definite integrals or by evaluating a definite integral of the absolute value of the difference of two functions.
- Volumes of solids with square, triangular, rectangle, or semicircular cross sections can be found using definite integrals and the area formulas for these shapes may be found by using definite integrals.
- The formula to find the length of a planar curve. *

Students will be able to:

- Determine the average value of a function using definite integrals.
- Determine values for positions and rates of change using definite integrals and problems involving rectilinear motion
- Interpret the meaning of a definite integral in accumulation problems.
- Determine net change using definite integrals in applied contexts.
- Calculate areas in the plane using the definite integrals, using functions of either x or y.
- Calculate volumes of solids with known cross-sections using definite integrals.
- Calculate volumes of solids or revolution using definite integrals using disc method or washer method for solids revolved around the x-axis, y-axis, or any other horizontal or vertical line in the plane.
- Determine the lengths of a curve in the plane defined by the function, using a definite integral. *
- Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems.

Resources

Core Text: *Calculus-Graphical, Numerical, Algebraic* (2012). Finney, Demana, et al. Prentice Hall

Suggested Resources: *Calculus* (2009) Hughes-Hallet, Gleason, et al, John Wiley & Sons, Inc.; khanacademy.org; desmos.com; collegeboard.org; graphing calculators

UNIT 7: Differential Equations

Summary and Rationale

In this unit, students will learn to set up and solve separable differential equations. Slope fields can be used to represent solution curves to a differential equation and build understanding that there are infinitely many general solutions to a differential equation, varying only by a constant of integration. Students can locate a unique solution relevant to a particular situation, provided they can locate a point on the solution curve. By writing and solving differential equations leading to models for exponential growth and decay and logistic growth, students build understanding of topics introduced in Algebra II and other courses.

Recommended Pacing

11 days

AP Big Ideas

Standard FUN - Functions

#	Outcomes
7.A	Interpret verbal statements of problems as differential equations involving a derivative expression.
7.B	Verify solutions to differential equations.
7.C	Estimate solutions to differential equations.
7.D	Determine general solutions to differential equations.
7.E	Determine particular solutions to differential equations.
7.F	Interpret the meaning of a differential equation and its variables in context.
7.G	Determine general and particular solutions for problems involving differential equations in context.
7.H	Interpret the meaning of the logistic growth model in context.

Instructional Focus

Unit Enduring Understandings

- Solving differential equations allows us to determine functions and develop models.

Unit Essential Questions

- What does a slope field represent?
- When there are multiple approaches, how should you choose the best method?
- How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?

Objectives

Students will know:

- Differential equations relate a function of an independent variable and the function's derivatives.
- Derivatives can be used to verify that a function is a solution to the differential equation.
- There may be infinitely many general solutions to a differential equation.
- Slope field is a graphical representation of a differential equation on a finite set of points in the plane.
- Slope fields provide information about the behavior of solutions to first order differential equations.
- Solutions to differential equations are functions or families of functions.
- Euler’s method provides a procedure for approximating a solution to a differential equation. *
- Some differential equations could be solved by separation of variables.
- Anti-differentiation can be used to find general solutions to differential equations.

- A general solution may describe infinitely many solutions to the differential equation. There is only one particular solution passing through a given point.
- Solutions to differential equations may be subject to domain restrictions.
- Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line and exponential growth and decay
- The model for exponential growth and decay that arises from the statement, "The rate of change of a quantity is proportional to the size of the quantity" is $\frac{dy}{dt} = ky$.
- The exponential growth and decay model, $\frac{dy}{dt} = ky$, with initial condition $y = y_0$ when $t = 0$, has solutions of the form $y = y_0 e^{kt}$.
- The model for logistic growth that arises from the statement, "The rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity" is $\frac{dy}{dt} = ky(a - y)$.*
- The logistic differential equation and initial conditions can be interpreted without solving the differential equation.*
- The carrying capacity of a logistic differential equation as the independent variable approaches infinity can be determined using the logistic growth model and initial conditions.*
- The value of the dependent variable in a logistic differential equation at the point when it is changing fastest can be determined using the logistic growth model and initial conditions.*

Students will be able to:

- Interpret verbal statements of problems as differential equations involving a derivative expression.
- Verify solutions to differential equations.
- Construct slope fields and interpret slope fields as visualizations of differential equations.
- Use technology to analyze slope fields and recognize a solution and its domain, e.g. Desmos.com.
- Estimate solutions to differential equations graphically using slope fields and numerically using Euler's Method.*
- Determine general and particular solutions to differential equations (for problems involving differential equations in context).
- Interpret the meaning of a differential equation and its variables in context.
- Interpret the meaning of the logistic growth model in context. *
- Determine the carrying capacity and when the value of the dependent variable in a logistic differential equation at the point when it is changing fastest graphically and algebraically.
- Apply appropriate mathematical rules or procedures, with and without technology.

Resources

Core Text: *Calculus-Graphical, Numerical, Algebraic* (2012). Finney, Demana, et al. Prentice Hall

Suggested Resources: *Calculus* (2009) Hughes-Hallet, Gleason, et al, John Wiley & Sons, Inc.; khanacademy.org; desmos.com; collegeboard.org; graphing calculators

UNIT 8: Infinite Sequences and Series *

Summary and Rationale

In this unit, students need to understand that a sum of infinitely many terms may converge to a finite value. They can develop intuition by exploring graphs, tables, and symbolic expressions for series that converge and diverge and for Taylor polynomials. Students should build connections to past learning, such as how evaluating improper integrals relates to the integral test or how using limiting cases of power series to represent continuous functions relates to differentiation and integration. Students who rely solely on memorizing a long list of tests and procedures typically find little success achieving a lasting conceptual understanding of series.

Recommended Pacing

18 days

AP Big Ideas

Standard LIM - Limits

#	Outcomes
7.A	Determine whether a series converges or diverges.
7.B	Approximate the sum of a series.
8.A	Represent a function at a point as a Taylor polynomial.
8.B	Approximate function values using a Taylor polynomial.
8.C	Determine the error bound associated with a Taylor polynomial approximation.
8.D	Determine the radius of convergence and interval of convergence for a power series.
8.E	Represent a function as a Taylor series or a Maclaurin series
8.F	Interpret Taylor series and Maclaurin series.
8.G	Represent a given function as a power series.

Instructional Focus

Unit Enduring Understandings

- Applying limits may allow us to determine the finite sum of infinitely many terms.
- Power series allow us to represent associated functions on an appropriate interval.

Unit Essential Questions

- How can the sum of infinitely many discrete terms be a finite value or represent a continuous function?
- When there are multiple approaches, how should you choose the best method?

Objectives

Students will know:

- The nth partial sum is defined as the sum of the first n terms of a series
- An infinite series converges to a real number S if and only if the limits of its sequence of partial sums exists and equals S.
- Geometric series is a series of the constant ratio between successive terms.
- The nth term test is a test only for divergence of a series.
- The integral test, comparison test, limit comparison test, and ratio test are methods to determine whether a series converges or diverges.
- The conditions that must be met before applying a test/rule.
- The alternating series test is a method to determine whether an alternating series converges.

- If a series converges absolutely, then it converges.
- If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value.
- Alternating series error bound can be used to bound how far a partial sum is from the value of the infinite series and to bound the error of a Taylor polynomial approximation to the value of a function.
- The coefficient of the nth degree term in a Taylor polynomial for a function f centered at $x = a$ is $\frac{f^{n}(a)}{n!}$.
- Many cases, as the nth degree of a Taylor polynomial increases, the nth degree polynomial will approach the original function over some interval.
- Taylor polynomials for a function f centered at $x = a$ can be used to approximate function values of f near $x = a$.
- The Lagrange error bound can be used to determine a maximum interval for the error of a Taylor polynomial approximation to a function.
- A power aries is a series of the form $\sum_{n=0}^{\infty} a_n(x - r)$.
- If a power series converges, it either converges at a single point or has an interval convergence.
- The radius of convergence of a power series can be used to identify an open interval on which the series converges, but it is necessary to test both endpoints of the interval to determine the interval convergence.
- If the power series has a positive radius of convergence, then the power series is a Taylor series of the function to which it converges over the open interval.
- The radius of convergence of a power series obtained by term-by-term differentiation or term-by-term integration is the same as the radius of convergence of the original power series.
- A Taylor polynomial for $f(x)$ is a partial sum of the Taylor series for $f(x)$.
- Using a known series, a power series for a given function can be derived using operations such as term by term differentiation or term by term integration, and by various methods (e.g. algebraic processes, substitutions or using properties of geometric series).

Students will be able to:

- Determine whether a series converges or diverges using the appropriate test/rule.
- Find the sum of a geometric series.
- Recognize, in addition to geometric series, common series such as the harmonic series, the alternating harmonic series and p-series.
- Determine if a series is absolutely convergent, conditionally convergent or divergent.
- Represent a function at a point as a Taylor polynomial.
- Represent a function as a Taylor series or a Maclaurin series.
- Interpret Taylor series or a Maclaurin series.
- Approximate function values using a Taylor polynomial.
- Construct the Maclaurin series of other functions using the Maclaurin series for $sinx$, $cosx$ and e^{x}.
- Determine the error bound associated with a Taylor polynomial approximation using Lagrange error bound or alternating series error bound, where applicable.
- Determine the radius of convergence and interval of convergence of a power series using the ratio test.
- Represent a given function as a power series.
- Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems.

Resources

Core Text: *Calculus-Graphical, Numerical, Algebraic* (2012). Finney, Demana, et al. Prentice Hall

Suggested Resources: *Calculus* (2009) Hughes-Hallet, Gleason, et al, John Wiley & Sons, Inc.; khanacademy.org;

desmos.com; collegeboard.org; graphing calculators

UNIT 9: Parametric Equations and Polar Coordinates and Vector Valued- Functions *

Summary and Rationale

In this unit, students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas. This unit should be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize

Recommended Pacing

10 days

AP Big Ideas

Standard CHA - Changes

#	Outcomes
3.G	Calculate derivatives of parametric functions.
3.H	Calculate derivatives of vector-valued functions.
5.D	Calculate areas of regions defined by polar curves using definite integrals.
6.B	Determine the length of a curve in the plane defined by parametric functions, using a definite integral.

Standard FUN - Functions

#	Outcomes
8.A	Determine a particular solution given a rate vector and initial conditions.
8.B	Determine values for positions and rates of change in problems involving planar motion.
3.G	Calculate derivatives of functions written in polar coordinates.

Instructional Focus

Unit Enduring Understandings

- Derivatives allow us to solve real-world problems involving rate of change.
- Definite integrals allow us to solve problems involving the accumulation of change over an interval.
- Solving an initial value problem allows us to determine an expression for the position of a particle moving in the plane.
- Recognizing opportunities to apply derivative rules can simplify differentiation.

Unit Essential Questions

- How can we model motion not constrained to a linear path?
- How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?

Objectives

Students will know:

- Methods for calculating derivatives of real value functions can be extended to parametric functions, vector-valued functions, and functions written in polar coordinates.

- Methods for calculating integrals of real value functions can be extended to parametric functions and vector-valued functions.
- Derivatives can be used to determine velocity, speed and acceleration for a particle moving along a curve in the clean defined using prometric or vector valued functions.
- For a particle in planar motion over an interval of time, the definite integral of the velocity vector represents the particle's displacement over the interval of time, from which we might determine its position
- For a curve given by a polar equation, $r = f(\theta)$, derivatives of r, x, and y with respect to θ, and first and 2nd derivatives of y with respect to x can provide information about the curve.
- Areas of regions bounded by polar curves can be calculated with definite integrals.

Students will be able to:

- Find the slope of the line tangent to a curve defined using parametric equations.
- Calculate (the first and second) derivatives of parametric functions, vector-valued functions and functions written in polar coordinates.
- Determine the length of a curve in the plane defined by parametric functions, using a definite integral.
- Determine a particular solution given a rate vector and initial conditions.
- Determine values for positions and rates of change and problems involving planar motion.
- Calculate areas or regions defined by polar curves using definite integrals.

Resources

Core Text: *Calculus-Graphical, Numerical, Algebraic* (2012). Finney, Demana, et al. Prentice Hall

Suggested Resources: *Calculus* (2009) Hughes-Hallet, Gleason, et al, John Wiley & Sons, Inc.; khanacademy.org; desmos.com; collegeboard.org; graphing calculators