



PISCATAWAY TOWNSHIP SCHOOLS

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Honors Calculus

Content Area: Mathematics

Grade Span: 11-12

Revised by: Tonya McGovern, mathematics teacher PHS

Approval Date: August 2021

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COURSE OVERVIEW

Description

This course is the last portion of an accelerated mathematics program for students who do not take an Advanced Placement Calculus course. In addition to mastering precalculus skills, and working with college placement practice tests, this course is designed to be an introduction to calculus material for students who will be taking a calculus course in college and does not serve as a complete background for a student who intends to take the Advanced Placement exam. A TI-83 graphing calculator that the school will provide is used throughout the course. Because this is an honors course the grades are weighted for GPA calculation.

Goals

Master precalculus topics, score high enough on a college placement test to start with a Calculus class in college and introduce, explore and investigate Calculus I level topics. In addition, the Mathematical Practices for AP Calculus (MPACs) capture important aspects of the work that mathematicians engage in, at the level of competence expected of Honors Calculus students. Embedding these practices in the study of calculus enables students to establish mathematical lines of reasoning and use them to apply mathematical concepts and tools to solve problems. The MPACs are not intended to be viewed as discrete items that can be checked off a list; rather, they are highly interrelated tools that should be utilized frequently and in diverse contexts. The MPACs are given below:

1. Reasoning with definitions and theorems.
2. Connecting concepts.
3. Implementing algebraic/computational processes.
4. Connecting multiple representations.
5. Building notation fluency.
6. Communicating

Scope and Sequence

Unit	Topic	Length (1 day = 80 minutes of instruction)
Unit 1	Library of functions	20 days
Unit 2	The Derivative	12 days
Unit 3	Short-cuts to Derivatives	12 days
Unit 4	Using the Derivative	20 days
Unit 5	The Definite Integral	12 days
Unit 6	Constructing Antiderivatives	12 days

Resources

Core Text: Calculus 5th Edition Single Variable (2009) Hughes-Hallet, Gleason, et al, John Wiley & Sons, Inc.

Suggested Resources:

<https://www.khanacademy.org/>
<https://www.calc-medic.com/>
<https://www.collegeboard.org/>
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<http://webspace.ship.edu/msrenault/GeoGebraCalculus/GeoGebraCalculusApplications.html>

ALL UNITS: MATHEMATICAL PRACTICES FOR HONORS CALCULUS

Summary and Rationale	
All units in the Honors Calculus curriculum have shared process standards that form the basis of the inquiries undertaken. These ideas, known as the Mathematical Practices for AP Calculus, are outlined below.	
State Standards	
MPAC 1: Reasoning with definitions and theorems	
	Cumulative Progress Indicator (CPI)
	Use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results.
	Confirm that hypotheses have been satisfied in order to apply the conclusion of a theorem.
	Apply definitions and theorems in the process of solving a problem.
	Interpret quantifiers in definitions and theorems (e.g., "for all," "there exists").
	Develop conjectures based on exploration with technology.
	Produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures.
MPAC 2: Connecting Concepts	
	Cumulative Progress Indicator (CPI)
	Relate the concept of a limit to all aspects of calculus.
	Use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems.
	Connect concepts to their visual representations with and without technology.
	Identify a common underlying structure in problems involving different contextual situations.
MPAC 3: Implementing algebraic/computational processes	
	Cumulative Progress Indicator (CPI)
	Select appropriate mathematical strategies.
	Sequence algebraic/computational procedures logically.
	Complete algebraic/computational processes correctly.
	Apply technology strategically to solve problems.
	Attend to precision graphically, numerically, analytically, and verbally and specify units of measure.
	Connect the results of algebraic/computational processes to the question asked.
MPAC 4: Connecting multiple representations	
	Cumulative Progress Indicator (CPI)
	Select Associate tables, graphs, and symbolic representations of functions.
	Develop concepts using graphical, symbolical, verbal, or numerical representations with and without technology.
	Identify how mathematical characteristics of functions are related in different representations.
	Extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values).
	Construct one representational form from another (e.g., a table from a graph or a graph from given information).
	Consider multiple representations (graphical, numerical, analytical, and verbal) of a function to select or construct a useful representation for solving a problem.

MPAC 5: Building notational fluency	
	Cumulative Progress Indicator (CPI)
	Know and use a variety of notations.
	Connect notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum).
	Connect notation to different representations (graphical, numerical, analytical, and verbal).
	Assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.
MPAC 6: Communicating	
	Cumulative Progress Indicator (CPI)
	Clearly present methods, reasoning, justifications, and conclusions.
	Use accurate and precise language and notation.
	Explain the meaning of expressions, notation, and results in terms of a context (including units).
	Explain the connections among concepts.
	Critically interpret and accurately report information provided by technology.
	Analyze, evaluate, and compare the reasoning of others.

UNIT 1: Library of functions

Summary and Rationale

Functions and graphs form the basis for understanding mathematics and applications. This unit introduces all the elementary functions to be used in the course. Although the functions are probably familiar, the graphical, numerical, verbal, and analytical approach to their analysis may be new. In addition, the limit is a fundamental concept in higher math. A theoretical understanding of the limit allows us to work with infinitesimally small values, building the bridge from estimated slopes and areas to the exact values found by applying derivatives and integrals. Students must have a solid, intuitive understanding of limits and be able to compute various limits, including one-sided, limits at infinity, and infinite limits.

Recommended Pacing

20 days

Instructional Focus

Unit Enduring Understandings

- Algebraic representation can be used to generalize patterns and relationships
- Patterns and relationships can be represented graphically, numerically, symbolically, or verbally
- Mathematical models can be used to describe and quantify physical relationships
- Equations that model real-world data allow you to make predictions about the future
- The concepts of a limit can be used to understand the behavior of functions
- Continuity is a key property of functions that is defined using limits
- There are many ways of evaluating a limit
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole

Unit Essential Questions

- What is the most effective way to solve a problem?
- What is the best answer to a problem?
- What is the best way to communicate mathematically?
- How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?
- How are patterns of change related to the behavior of functions?
- How can we use mathematical models to describe physical relationships?

Objectives

Students will know:

- point-slope form and its derivation
- definition of an even function and an odd function
- definition of an inverse of a function
- properties of logarithms
- the difference between solving a linear equation or inequality and nonlinear equation or inequality
- multiple representations of functions
- categories of functions: including linear, absolute value, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric
- the absolute value of a function will change only the negative values of $f(x)$ to its opposite value
- domain and range in interval notation
- evaluating functions including composite functions
- language of limits, include notation of one-sided limits and two-sided limits
- the limit of a some functions may be found by using algebraic manipulation
- asymptotic and unbounded behavior of functions can be explained and described using limits
- properties of limits

- types of discontinuities
- definition of continuity at a point
- polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains
- limit of the difference quotient is the definition of a derivative
- limits can be used to describe the behavior of functions for large numbers
- how to use the TI-83 Graphing Calculator to solve problems, experiment, interpret results, and support conclusions
- **Vocabulary:** Asymptote, logarithmic properties, inclusive, exclusive, limit, one-sided limit, two-sided limit, limit properties, continuity at a point, continuous function, tangent line, secant line, difference quotient

Students will be able to:

- write an equation and sketch a graph of a linear, quadratic, polynomial or exponential function given specific information
- identify the relationships between parallel lines, perpendicular lines, and slopes
- identify the domain and range of a function using its graph or equation
- write and evaluate compositions of two functions
- identify a one-to-one function
- determine the algebraic and graphical representation of a function and its inverse
- apply the properties of logarithms
- generate the equations of graphs for the trigonometric functions that model real-world scenarios
- work with tables, graphs and properties in order to estimate the limit of a function at a point

Resources

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UNIT 2: The Derivative

Summary and Rationale

Using derivatives to describe the rate of change of one variable with respect to another variable allows students to understand change in a variety of contexts. With their understanding of functions, students will recognize that the slopes of the tangents at the given points represent a relationship between the two quantities. Students will call this function the derivative of a function. The derivative is the key to modeling instantaneous change. Students should be able to interpret the first and second derivative and determine differentiability.

Recommended Pacing

12 days

Instructional Focus

Unit Enduring Understandings

- The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies
- A function's derivative, which is itself a function, can be used to understand the behavior of the function
- There are many ways of evaluating the derivative
- Relationships can be represented graphically, numerically, analytically, or verbally

Unit Essential Questions

- How does the concept of a limit lead to the derivative?
- What is a derivative and how does it differ in various situations?
- What can you predict about f given f' ?
- When there are multiple approaches, how should you choose the best method?
- What are the advantages of having different ways to represent a derivative?

Objectives

Students will know:

- evaluating derivatives using the difference quotient definition and the derivative at a point definition
- the difference quotient represents the slope of a secant or average rate of change over a given interval
- the limit of the difference quotient represents the slope of a tangent or instantaneous rate of change at a given point, also called the derivative at that point
- the relationships between the graph of $f(x)$ and $f'(x)$ and $f''(x)$
- how to determine where a function is and is not differentiable
- slopes of linear functions (local linearity) represent an estimation for the slope of a curve at a given point
- derivative as a rate of change
- the derivative at a point can be estimated from information given in tables or graphs
- if a function is increasing then its derivative is greater than zero and if a function is decreasing then its derivative is less than zero
- differentiability implies continuity
- the derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable
- speed is the absolute value of velocity
- velocity is the derivative of position and acceleration is the derivative of velocity
- **Vocabulary:** first derivative, second derivative, tangent line, difference quotient, differentiability, continuity, discontinuity, cusp, vertical tangent, speed, average rate of change, average velocity, instantaneous velocity, position, displacement, acceleration, local linearity, infinitesimal

Students will be able to:

- find the average rate of change on a given interval
- find the instantaneous rate of change at a given point
- discern where a function is and is not differentiable

- distinguish between corners, cusps, discontinuities, and vertical tangents
- graph the derivative function from data
- calculate the first and second derivatives using the derivative definition (limit of a difference quotient)
- use derivatives to analyze straight line motion and solve other problems involving rates of change
- estimate the derivative at a given point using a table of values or the graph of f
- recognize the connection between differentiability and continuity
- determine whether a function is differentiable at a given point by identifying a corner, cusp, vertical tangent or discontinuity in the graph of f
- graph f' using the graph of f and vice versa
- use a graphing calculator to solve problems, experiment, interpret results, and support conclusions

Resources

Core Text: Calculus 5th Edition Single Variable (2009) Hughes-Hallet, Gleason, et al, John Wiley & Sons, Inc.

Suggested Resources:

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UNIT 3: Short-cuts to Derivatives

Summary and Rationale
<p>Using derivatives to describe the rate of change of one variable with respect to another variable allows students to understand change in a variety of contexts. With their understanding of functions, students will recognize that the slopes of the tangents at the given points represent a relationship between the two quantities. Students will call this function the derivative of a function. The derivative is the key to modeling instantaneous change. Students should be able to estimate derivatives from tables and graphs, and apply various derivative rules and properties.</p>
Recommended Pacing
12 days
Instructional Focus
Unit Enduring Understandings
<ul style="list-style-type: none"> • The type of function that exists determines what type of shortcut to apply when finding the derivative • The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies • A function's derivative, which is itself a function, can be used to understand the behavior of the function • There are many ways of evaluating the derivative • Relationships can be represented graphically, numerically, analytically, or verbally
Unit Essential Questions
<ul style="list-style-type: none"> • What type of function does a given equation represent? • What is the most effective way to solve a problem? • What are the advantages of having different ways to represent a derivative? • What is a derivative and how does it differ in various situations? • What can you predict about given ' or ''? • When there are multiple approaches, how should you choose the best method?
Objectives
<p>Students will know:</p> <ul style="list-style-type: none"> • specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, and trigonometric • chain rule is used for composition of functions • Leibniz notation is necessary to compute implicit differentiation • Local linearization describes the behavior of a function near a point and approximates the rate at which a curve moves at a specific point • If a function has a k between $f(a)$ and $f(b)$ then there is at least 1 number c such that $a < c < b$ and $f(c) = k$. • Between two points of a continuous function there is at least 1 ordered pair that's derivative is the same as the slope of the secant line • Vocabulary: power rule, product rule, quotient rule, trig ladder mnemonic device [sin,cos,-sin,-cos], composition, decomposition, chain rule, Leibniz notation, implicit differentiation, local linearization, Intermediate Value theorem, Mean Value theorem <p>Students will be able to:</p> <ul style="list-style-type: none"> • memorize and apply the shortcuts to differentiation for the first 3 basic trig functions and polynomial, exponential and logarithmic functions • recognize the type of function so the correct shortcut to differentiation can be applied • recognize that Euler's number, e, to the x power is the only function that is its own derivative • decompose functions by identifying the inside and outside function before using the chain rule • make predictions using the Intermediate Value and Mean Value theorems • use a graphing calculator to solve problems, experiment, interpret results, and support conclusions

Resources

Core Text: Calculus 5th Edition Single Variable (2009) Hughes-Hallet, Gleason, et al, John Wiley & Sons, Inc.

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UNIT 4: Using the Derivative

Summary and Rationale

Students should see how to use the derivative in real life scenarios. Differential calculus is a powerful problem-solving tool precisely because of its usefulness for analyzing functions. Finding maximum and minimum values of functions, called optimization, is an important issue used in multiple business and scientific models. Students in this unit will be using the derivative in applications that include finding the slope of a tangent line to a graph at a point and analyzing the graph of a function. They will sketch the graph of a function by determining where it is increasing or decreasing and finding inflection points and extreme values. In addition, students should understand and be able to apply common Calculus theorems, and be familiar with a variety of real world applications, including related rates, optimization, and using growth and decay models.

Recommended Pacing

20 days

Instructional Focus

Unit Enduring Understandings

- Calculus is a collection of powerful ideas; not a set of rules, formulas and procedures
- To learn calculus requires focus on the understanding of a few big ideas, not merely memorizing techniques
- A function's derivative, which is itself a function, can be used to understand the behavior of the function
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole
- Mathematical models can be used to describe and quantify physical relationships
- Physical models can be used to clarify mathematical relationships

Unit Essential Questions

- How would a business determine maximize profits and minimal costs?
- How does the rate at which a liquid is flowing affect the volume of the liquid?
- How can we use mathematical models to describe physical relationships?
- How can we use physical models to clarify mathematical relationships?
- How do we measure how something changes with respect to time?
- How can we use derivatives to solve problems?
- What can you predict about $f'(c)$ given $f(c)$?

Objectives

Students will know:

- the Intermediate Value theorem states if a function has a k between $f(a)$ and $f(b)$ then there is at least 1 number c such that $a < c < b$ and $f(c) = k$
- the Mean Value theorem states that between two points of a continuous function there is at least 1 ordered pair that's derivative is the same as the slope of the secant line between the two endpoints
- the Extreme Value theorem states there is always a global max and min on a closed interval for a continuous function
- increasing functions have a positive derivative and decreasing functions have a negative derivative
- if a function's first derivative is increasing, then the function is concave up and if the function's first derivative is decreasing, then the function is concave down
- concave up functions have a positive second derivative and concave down functions have a negative second derivative
- local extrema are found using critical points.
- modeling and optimization allow someone to make accurate predictions for fixed data to create maximum profit or minimum cost predictions
- local linearization is writing the equation of the tangent line near a point
- related rates are a differential equation

- L'Hopital's rule can be used to find limits of indeterminate cases
- **Vocabulary:** Intermediate Value theorem, Mean Value theorem, Extreme Value theorem, first and second derivative tests, concavity, inflection, local and global extrema, local maximums and minimums, optimization, related rates, differential equation, indeterminate, L'Hopital's rule

Students will be able to:

- find derivatives using implicit differentiation
- interpret the meaning of a derivative in the context of the problem
- determine the local or global extrema of a function
- find the intervals on which a function is increasing or decreasing, and concave up or down
- apply common Calculus theorems to make accurate predictions
- determine the concavity of a function and locate points of inflection by analyzing the second derivative
- graph $f(x)$ using information about $f'(x)$ and $f''(x)$
- connect f to f' to f'' graphically by looking at 3 graphs and determining which is f , f' and f''
- find the local linearizations of a function near given points
- solve application problems by finding minimum or maximum values of functions
- estimate the change in a function using differential equations
- solve related rates problems
- identify limit problems that L'Hopital's rule can be applied to
- use a graphing calculator to solve problems, experiment, interpret results, and support conclusions

Resources

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[ets.html](http://webspace.ship.edu/msrenault/GeoGebraCalculus/GeoGebraCalculusApplications.html)

Unit 5: The Definite Integral

Summary and Rationale

Integrals are used in a wide variety of practical and theoretical applications. Students should have a practical understanding of the definition of a definite integral involving a Riemann sum, be able to approximate a definite integral using different methods, and be able to compute definite integrals using geometry. They should be familiar with basic techniques of integration and properties of integrals. It is critical to understanding that students graph the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus.

Recommended Pacing

12 days

Instructional Focus

Unit Enduring Understandings

- The type of function that exists determines what type of shortcut to apply when finding the antiderivative
- Antidifferentiation is the inverse process of differentiation
- The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies
- The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole
- Mathematical models can be used to describe and quantify physical relationships
- Physical models can be used to clarify mathematical relationships

Unit Essential Questions

- What does a definite integral represent?
- How can we use mathematical models to describe physical relationships?
- How can we use physical models to clarify mathematical relationships?
- When there are multiple approaches, how should you choose the best method?
- When will a process yield an estimated answer versus an exact answer?

Objectives

Students will know:

- the antiderivative of a function is a function if the derivative of the function is the function
- terminology and notation of a definite integral
- differentiation rules provide the foundation for finding antiderivatives
- a Riemann sum, which requires a partition of an interval, is the sum of products, each of which is the value of a function at a point in a subinterval multiplied by the length of that subinterval of the partition
- the information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral
- a left hand Reimann sum is an underestimate for decreasing functions and an overestimate for increasing functions
- a right hand Reimann sum is an overestimate for decreasing functions and an underestimate for increasing functions
- a Riemann sum approximation may or may not be an exact value of a definite integral depending of the function that is being integrated by area under the curve
- a midpoint Reimann sum or a trapezoidal Riemann sum will likely be more accurate than a left or right hand Reimann sum
- definite integrals can be approximated for functions that are represented graphically, numerically, analytically, and verbally
- the Fundamental Theorem of Calculus can only be used if the integrand is a continuous function

- properties of definite integrals
- the formula for the average value of a function over a closed interval
- the relationship between area under a velocity curve and distance traveled
- **Vocabulary:** antiderivative, integrand, interval, sigma, summation, definite integral, left hand, right hand, trapezoidal and midpoint Riemann sums, average value of a function, distance travelled, displacement, First Fundamental Theorem of Calculus

Students will be able to:

- recognize antiderivatives of basic functions
- interpret the definite integral as the limit of a Riemann Sum
- express the limit of a Riemann sum in integral notation
- express the area under a curve as a definite integral
- approximate a definite integral by using left hand, right hand, trapezoidal and midpoint Riemann sums
- interpret the area under the graph as a net accumulation of a rate of change
- calculate a definite integral using areas and properties of definite integrals
- apply the Fundamental Theorem of calculus by analyzing functions defined by an integral and by constructing antiderivatives
- find the average value of a function over a closed interval
- use a graphing calculator to solve problems, experiment, interpret results, and support conclusions - include how to evaluate a definite integral

Resources

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Unit 6: Constructing Antiderivatives

Summary and Rationale

It is one thing to do Calculus but it is likely to have a more profound effect on success to understand it. The process of how to integrate can be mastered but it is imperative to know how to apply integration. To be accomplished working with an antiderivative, one must focus on what the integration of an integrand will produce. Integrals are used in a variety of applications to model physical, biological, or economic situations so to construct an antiderivative will allow a person to apply Calculus topics to real world life applications.

Recommended Pacing

12 days

Instructional Focus

Unit Enduring Understandings

- The force on an object affects acceleration and changes velocity
- Antidifferentiation is the inverse process of differentiation
- The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole
- Mathematical models can be used to describe and quantify physical relationships
- Physical models can be used to clarify mathematical relationships

Unit Essential Questions

- How do the velocity and the position of an object vary with time?
- How does acceleration come about?
- How does velocity change?
- What does a definite integral represent compared to an indefinite integral?
- When there are multiple approaches, how should you choose the best method?
- How can we use mathematical models to describe physical relationships?
- How can we use physical models to clarify mathematical relationships?

Objectives

Students will know:

- principle of inertia: an object traveling undisturbed at a constant velocity in a straight line will continue in this motion indefinitely and as a result one cannot distinguish between being at rest and moving with constant velocity in a straight line therefore motion should be based on change in velocity rather than change of position
- the Fundamental Theorem of Calculus can only be used if the integrand is a continuous function
- given f is a derivative, the function f has infinite antiderivatives
- through a vertical shift only, there are infinite functions with the same derivative
- the indefinite integral of a function creates a family of functions
- if $f'(x) = 0$ on an interval. then $f(x) = C$ on this interval for some constant C
- where f' is above the x-axis, f is increasing
- where f'' is above the x-axis, f' is increasing and f is concave up
- where f' is below the x-axis, f is decreasing
- where f'' is below the x-axis, f' is decreasing and f is concave down
- which shortcut formula to use for different antiderivatives
- how to recognize when integration by U substitution or integration by parts is necessary
- the antiderivative of acceleration is velocity and the antiderivative of velocity is position

- **Vocabulary:** indefinite integral, unbounded, family of functions, constant acceleration, differential equation, general solution, initial value problem, second Fundamental Theorem of Calculus, integration by U substitution, integration by parts, inertia, force

Students will be able to:

- use the definite integral to construct antiderivatives
- identify what type of function the integrand is to be able to apply the correct shortcut rule for integration
- calculate the derivative of an integral including how to integrate by U substitution and integrate by parts
- derive the projectile motion formula for Physics using Calculus
- predict time and height of an object with different parameters given uniformly accelerated motion
- find the solution to an initial value problem
- use a graphing calculator to solve problems, experiment, interpret results, and support conclusions

Resources

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