PISCATAWAY TOWNSHIP SCHOOLS

Dr. Frank Ranelli
Superintendent of Schools

Dr. William Baskerville
Assistant Superintendent

Applied Statistics

Content Area: **Mathematics**

Grade Span: **11-12**

Revised by: **Christopher Mango and H. Jermaine Robinson**

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Piscataway Township Schools
1515 Stelton Road
Piscataway, NJ 08854-1332
732 572-2289, ext. 2561
Fax 732 572-1540
www.piscatawayschools.org

COURSE OVERVIEW

Description

The general purpose of Applied Statistics is to learn about real-world problems pertaining to statistics and probability by exploring four broad conceptual themes: exploring data, sampling and experimenting, anticipating patterns, and drawing inferences. These themes will be the motivators for the various topics, assignments, and assessments that will be encountered in this class.

Goals

In addition to the course-specific content standards, skills, and concepts of Applied Statistics, this class also seeks to meet the Standards for Mathematical Practice promoted in the Common Core State Standards Initiative. These goals include generally applied best practices for learning mathematics, such as understanding the nature of proof and having a productive disposition towards the subject, and broadly apply across all mathematics courses.

The eight Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Scope and Sequence

Unit	Topic	Length
Unit 1	Ch.1 - Introduction to Statistics	8 days
Unit 2	Ch.2 - Descriptive Statistics	11 days
Unit 3	Ch.9 - Correlation and Regression	7 days
Unit 4	Ch.3 - Probability and Combinatorics	14 days
Unit 5	Ch.4 - Discrete Probability Distributions	6 days
Unit 6	Ch.5 - Normal Probability Distributions	9 days
Unit 7	Ch.6 - Estimation and Confidence Intervals	12 days
Unit 8	Ch.7 - One-Sample Hypothesis Testing	10 days

Resources

Core Text: Elementary Statistics Picturing The World (5th ed.) by Ron Larson and Betsy Farber

UNIT 1: Ch.1 - Introduction to Statistics

Summary and Rationale

In this unit, students will explore the overview of statistics, such as quantitative and qualitative data categorization, discovering the difference and importance relationship between descriptive and inferential statistics, and population vs. sample in drawing conclusions about data collections. Students will discover the components of experimental design, such as observational study, experiment and its control/treatment groups, placebo effect, confounding, double-blind study, blocking, and bias. Students will learn how various types of bias (non-response, sampling, response, loaded questions, voluntary response, vague wording, etc.) can affect data collection and results. Students will learn the various methods of sampling techniques and which ones are the most effective depending upon the data collection. The sampling techniques include simple random sample (SRS), systematic, stratified, cluster, convenience, and multistage sampling. Moreover, students will learn about and differentiate between sampling errors and non-sampling errors. Additionally, students will make connections about misleading statistics in learning the comparison of correlation and causation using several statement and caption examples. By the end of this chapter, students will be able to apply what they learn by collecting qualitative and quantitative data and using this chapter's vocabulary to organize, describe, and analyze their studies.

Recommended Pacing

8 days

State Standards

Standard S-ID.B: Summarize, represent, and interpret data on two categorical and quantitative variables

CPI #	Cumulative Progress Indicator (CPI)
5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Standard S-ID.C: Interpret linear models

CPI #	Cumulative Progress Indicator (CPI)
9	Distinguish between correlation and causation.

Standard S-IC.A: Understand and evaluate random processes underlying statistical experiments

CPI #	Cumulative Progress Indicator (CPI)
1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

Standard S-IC.B Make inferences and justify conclusions from sample surveys, experiments, and observational studies

CPI #	Cumulative Progress Indicator (CPI)
3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4	Use data from a sample survey to estimate a population mean or proportion.

Instructional Focus

Unit Enduring Understandings

- Statistics is collecting, organizing, analyzing, and interpreting data in order to make a decision or conclusion
- Data collection can be either quantitative or qualitative

- Statistical data can have misleading data and or biases
- Correlation does not imply causation
- Some sampling techniques may not fit well for interpreting data
- Experimental design contains a control group, randomization, and replication

Unit Essential Questions

- What are the similarities and differences between population and sample?
- How can data be categorized?
- What are the similarities and differences between descriptive statistics and inferential statistics?
- Which sampling techniques are helpful (or ineffective) for data collection? Why?
- What kinds of bias can lead to misleading data collection? Why?

Objectives

Students will know:

- The difference between quantitative and qualitative data
- The difference between population and sample
- Various sampling techniques
- Various biases
- The components of experimental design
- The four levels of measurement in data

Students will be able to:

- Define and know examples of various types of data
- Distinguish between correlation and causation
- Find and identify bias in statistical data
- Define and discover various sampling techniques
- Create surveys and gather data

Additional Resources

Unit Resources: “Elementary Statistics” textbook and supplementary handouts

UNIT 2: Ch.2 - Descriptive Statistics

Summary and Rationale

In this unit, students will continue to explore descriptive statistics, such as frequency distributions, class widths, and cumulative frequency distributions. Students will present data collections in the form of histograms, stem-and-leaf plots, scatter plots, dots plots, and pie charts. Furthermore, students will learn to identify, compute, and analyze statistical data by finding their measures of central tendency (mean, median, and mode), weighted averages, skews in data or distribution shapes, measures of variation, variances and standard deviations for populations and samples, measures of position, interquartile ranges, outliers, five-number summaries, box-and-whiskers plots, percentiles, standardized scores (Z-scores), centers, shapes, and spreads.

Recommended Pacing

11 days

State Standards

Standard S-ID.A: Summarize, represent, and interpret data on a single count or measurement variable

CPI #	Cumulative Progress Indicator (CPI)
1	Represent data with plots on the real number line (dot plots, histograms, and box plots).
2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Standard S-ID.B: Summarize, represent, and interpret data on two categorical and quantitative variables

CPI #	Cumulative Progress Indicator (CPI)
5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

Standard S-IC.B: Make inferences and justify conclusions from sample surveys, experiments, and observational studies

CPI #	Cumulative Progress Indicator (CPI)
6	Evaluate reports based on data.

Instructional Focus

Unit Enduring Understandings

- Presenting data in various plots and charts
- Statistical analyzation of data
- Statistical trends in data

Unit Essential Questions

- What data plot or chart is right for data collection?
- How is data analyzed?

Objectives

Students will know:

- Frequency distribution of data
- Center, shape, and spread of data

- Measures of central tendency, variation, and position

Students will be able to:

- Create frequency distributions and other graphs, and apply them to data sets
- Compute, compare, and analyze measures of central tendency, variation, and position
- Create box-and-whiskers plots and find quartiles and percentiles

Additional Resources

Unit Resources: “Elementary Statistics” textbook and supplementary handouts

UNIT 3: Ch.9 - Correlation and Regression

Summary and Rationale

In this unit, students will explore the foundations of a scatter plot by finding and analyzing its correlation, correlation coefficient, covariance, strength, direction, least-squares regression line, and residual plot. Students will be able to make predictions about scatter plot data through the concepts of interpolation and extrapolation. By the end of this chapter, students will have a greater understanding of patterns that fit collections of data.

Recommended Pacing

7 days

State Standards

Standard S-ID.B: Summarize, represent, and interpret data on two categorical and quantitative variables

CPI #	Cumulative Progress Indicator (CPI)
6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals, including by using technology. c. Fit a linear function for a scatter plot that suggests a linear association.

Standard S-ID.C: Interpret linear models

CPI #	Cumulative Progress Indicator (CPI)
7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model based on the data.
8	Compute and interpret the correlation coefficient of a linear fit.

Instructional Focus

Unit Enduring Understandings

- Finding trends and making interpretations from data and correlation coefficients
- Relationship between residual plots and least-squares regression lines

Unit Essential Questions

- How does the use of statistical analysis help with analyzing scientific data?
- Does a line of best fit always represent the data? Why or why not?
- What is the meaning of a correlation's strength and direction?

Objectives

Students will know:

- Covariance, correlation, and correlation coefficient
- Least-squares regression line and residual plots
- Interpolation is a safer predictor of missing values than extrapolation

Students will be able to:

- Distinguish between interpolation and extrapolation of data
- Compute a least-squares regression line using raw data
- Create a residual plot to further analyze a collection of data

Additional Resources

Unit Resources: "Elementary Statistics" textbook and supplementary handouts

UNIT 4: Ch.3 - Probability and Combinatorics

Summary and Rationale

In this unit, students will explore the foundations of probability, such as the fundamental counting principle, independence, disjoint events, and conditional probability. Students will create visual models of probability applications with two-way frequency tables, tree diagrams, and Venn diagrams. Additionally, students will make connections between probability and statistics through combinatorics: using factorials, combinations, permutations, Pascal's triangle, binomial theorem, and isomorphisms, students will be able to transition into the more advanced statistical topics in this course, such as working with binomial and normal distributions.

Recommended Pacing

14 days

State Standards

Standard S-CP.A: Understand independence and conditional probability and use them to interpret data

CPI #	Cumulative Progress Indicator (CPI)
2	Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3	Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Standard S-CP.B: Use the rules of probability to compute probabilities of compound events in a uniform probability model

CPI #	Cumulative Progress Indicator (CPI)
6	Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model.
7	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
8	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
9	Use permutations and combinations to compute probabilities of compound events and solve problems.

Instructional Focus

Unit Enduring Understandings

- Being able to compute the likelihood of events occurring has important, real-world applications
- Probability allows us to make predictions and informed decisions
- Repeatedly performing trials of events can help us understand probability, by collecting and analyzing data
- Probability scenarios can be represented, computed, and analyzed in a variety of ways
- Probability and combinatorics can be connected to statistics and data distributions

Unit Essential Questions

- What influences the probability that a given event will occur?
- What are the differences, and how can we verify, between theoretical and experimental probabilities; between with replacement and without replacement; between events that are disjoint and not disjoint; between events that are independent and not independent; between basic and conditional probabilities?

Objectives

Students will know:

- Basic probability notation and vocabulary
- Relationships between probability, combinatorics, and statistics
- Real-world applications involving probability and combinatorics

Students will be able to:

- Create and explain probability fractions
- Organize and represent applications using two-way frequency tables, tree diagrams, and Venn diagrams
- Recognize and evaluate problems that incorporate replacement, unions, intersections, and independence
- Compute and compare basic and conditional probabilities
- Compute and compare combinations and permutations
- Apply Pascal's triangle and factorials in relevant contexts

Additional Resources

Unit Resources: "Elementary Statistics" textbook and supplementary handouts

Unit 5: Ch.4 - Discrete Probability Distributions

Summary and Rationale

In this unit, students will explore discrete probability distributions and how to apply them to real-life situations. Besides concepts of probability distribution, students will learn about discrete and continuous random variables, the mean (expected value) and standard deviation of a discrete random variable, and how to combine discrete random variables. Furthermore, this chapter will cover concepts and real-life applications about binomial and geometric experiments and their properties and conditions.

Recommended Pacing

6 days

State Standards

Standard S-MD.A: Calculate expected values and use them to solve problems

CPI #	Cumulative Progress Indicator (CPI)
1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

Standard S-MD.B: Use probability to evaluate outcomes of decisions

CPI #	Cumulative Progress Indicator (CPI)
5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast food restaurant. b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Instructional Focus

Unit Enduring Understandings

- The use of probability will help to predict outcomes
- The similarities and differences between discrete random variables and continuous random variables
- Expected value will help to predict when event outcomes are (or are not) in your favor
- Understanding the relationship between combinatorics and binomial probability

Unit Essential Questions

- What is the expected value of a probabilistic event, and how is it calculated?
- How do we know whether to categorize a random variable as discrete or continuous?

Objectives

Students will know:

- Random variables
- Combining random variables
- Examples of discrete and continuous random variables
- Expected value
- The criteria for a binomial experiment
- Binomial and geometric probability distributions

Students will be able to:

- Compute expected value
- Create histograms and curves related to discrete and continuous random variables
- Calculate probabilities related to binomial and geometric distributions

Additional Resources

Unit Resources: “Elementary Statistics” textbook and supplementary handouts

Unit 6: Ch.5 - Normal Probability Distributions

Summary and Rationale

In this unit, students will learn to recognize normal distributions (also known as bell-shaped curves) and how to use their properties in real-life applications. Certain conditions must be met (or assumed) before concluding that a data set's distribution is normal, and sets of normally-distributed data have unique and conveniently predictable properties. Students will learn about the Empirical Rule (also known as the 68-95-99.7 Rule), and how raw scores, population means, population standard deviations, and standardized scores (also known as Z-scores) play a role in computing and analyzing normal probability distributions. The Z-table, areas under normal curves, sampling distributions, and the Central Limit Theorem are among additional topics that will be explored in this chapter.

Recommended Pacing

9 days

State Standards

Standard S-ID.A: A. Summarize, represent, and interpret data on a single count or measurement variable

CPI #	Cumulative Progress Indicator (CPI)
4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Instructional Focus

Unit Enduring Understandings

- The normal distribution can be used to model specific sets of data, given ideal conditions
- A sampling distribution is a probability distribution based on many samples from the same population
- The Central Limit Theorem maps sampling distributions onto an approximately normal distribution

Unit Essential Questions

- What are some applications of the normal distribution?
- What is a sampling distribution, and how does it differ from a sample?
- What is the Central Limit Theorem, and what is the value of being able to apply it to a study?

Objectives

Students will know:

- Properties of a normal distribution
- The formula for computing Z-score
- The Central Limit Theorem

Students will be able to:

- Recognize and apply Empirical Rule scenarios
- Read and use a Z-table
- Compute and interpret Z-scores
- Create and gather data to analyze sampling distributions

Additional Resources

Unit Resources: "Elementary Statistics" textbook and supplementary handouts

Unit 7: Ch.6 - Estimation and Confidence Intervals

Summary and Rationale

In this unit, students will utilize inferential statistics to make predictions and draw conclusions about populations, based on sample data. Students will learn how to estimate a population mean when the population standard deviation is known (through the normal distribution and the Z-table), as well as when the population standard deviation is unknown (through the Student t-distribution, the t-table, and degrees of freedom). Besides creating confidence intervals for the two aforementioned scenarios regarding means, students will also create confidence intervals with respect to proportions. Inferential vocabulary - point estimate, biased estimator, standard error, margin of error, and confidence level - will be introduced, as well as relevant formulas for confidence intervals.

Recommended Pacing

12 days

State Standards

Standard S-IC.A: Understand and evaluate random processes underlying statistical experiments

CPI #	Cumulative Progress Indicator (CPI)
1	Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Standard S-IC.B: Make inferences and justify conclusions from sample surveys, experiments, and observational studies

CPI #	Cumulative Progress Indicator (CPI)
3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6	Evaluate reports based on data.

Instructional Focus

Unit Enduring Understandings

- Inferential statistics are tools used for estimation and analysis
- Both the normal distribution and t-distributions can be used to model data sets
- Under the right conditions, confidence intervals can effectively estimate unknown population parameters

Unit Essential Questions

- What is the purpose of a confidence interval?
- What are the similarities and differences between descriptive statistics and inferential statistics?
- What are the similarities and differences between Z-scores and t-scores?
- What are the similarities and differences between Z-tables and t-tables?
- What are the similarities and differences between tests for means and tests for proportions?

Objectives

Students will know:

- New vocabulary surrounding inferential statistics
- The formulas for constructing confidence intervals (Z and t)

- Similarities and differences between means and proportions

Students will be able to:

- Read and use a t-table
- Draw inferences about populations, based on sample data
- Gather the information required to construct confidence intervals (Z and t)
- Compute and analyze confidence intervals (Z and t)

Additional Resources

Unit Resources: “Elementary Statistics” textbook and supplementary handouts

Unit 8: Ch.7 - One-Sample Hypothesis Testing

Summary and Rationale

In this unit, students will continue drawing inferences with respect to sample data, but will also learn how to lay out initial hypotheses that formalize statistical claims. The construction of hypothesis tests, from deciding on null and alternative hypotheses to using appropriate notation and inequality symbols, is the primary focus of this chapter. Students will learn about rejection regions and p-values, as well as what it means to reject the null hypothesis and either be correct or incorrect about that decision. Significance levels, Type 1 and Type 2 errors, alpha values, beta values, and the power of a statistical test will all be explored in this unit of Applied Statistics.

Recommended Pacing

10 days

State Standards

Standard S-IC.A: Understand and evaluate random processes underlying statistical experiments

CPI #	Cumulative Progress Indicator (CPI)
1	Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Standard S-IC.B: Make inferences and justify conclusions from sample surveys, experiments, and observational studies

CPI #	Cumulative Progress Indicator (CPI)
3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6	Evaluate reports based on data.

Instructional Focus

Unit Enduring Understandings

- Rejection regions and p-values are ways to evaluate the null hypothesis of a statistical test
- The adjustment of a confidence level or significance level can affect a rejection decision
- Inferential statistics are tools used for validating unknown population parameters

Unit Essential Questions

- What is the purpose of a hypothesis test?
- What is the significance of rejecting the null hypothesis?
- How do Type 1 error and Type 2 error factor into the risks of drawing incorrect inferences?

Objectives

Students will know:

- Characteristics and notation of null and alternative hypotheses
- Error types, significance levels, alpha, beta, and power
- Rejection regions, p-values, and what it means to reject the null hypothesis

Students will be able to:

- Construct and interpret null and alternative hypotheses based on applications
- Perform hypothesis tests and analyze conclusions
- Draw inferences about populations, based on sample data

Additional Resources

Unit Resources: “Elementary Statistics” textbook and supplementary handouts